

# Micro Genetic Algorithm Based Optimal Gate Positioning in Injection Molding Design

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## Abstract

The paper deals with the optimization of runner system in injection molding design. The design objective is to locate gate positions by minimizing both maximum injection pressure at the injection port and maximum pressure difference among all the gates on a product with constraints on shear stress and/or weld-line. The analysis of filling process is conducted by a finite element based program for polymer flow. Micro genetic algorithm (mGA) is used as a global optimization tool due to the nature of inherent nonlinearity in flow analysis. Four different design applications in injection molds are explored to examine the proposed design strategies. The paper shows the effectiveness of mGA in the context of optimization of runner system in injection molding design.

*Keywords:* Micro genetic algorithm; Design optimization; Filling injection mold

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## 1. Introduction

Injection molding process has been recognized as one of the most efficient manufacturing technologies since high performance polymer materials can be utilized to accurately manufacture a product with complicated shape (Chiang, et al., 1991; Chang and Yang, 2001; Himasekhar, et al., 1992; Kwon and Park, 2004). Also, the demand on injection molded products such as from conventional plastic goods to micro optical devices is being dramatically increased over the recent years (Piotter, et al., 2001; Kang, et al., 2000). In general, the injection mold process is initiated by the filling stage where the polymer materials fill into a cavity under the injection temperature. After the cavity is completely filled, the post-filling stage, that is, the packing stage is

conducted to be additionally filled with the high pressure polymer, thereby resulting in the avoidance of material shrinkage. Subsequently, the cooling stage is required for a molded product to be ejected without any deformation. It is important to accommodate the molding conditions in the filling stage since it is the first stage in the overall injection molding design (Zhou and D. Li, 2001). After that, one can successfully expect more improved molding conditions during post-filling stages such as packing, cooling stages. The paper deals with optimal conditions of the filling injection molding design in which the flow pattern and pressure for the polymer materials to be filled through gates of a runner are of significant. That is, one of design requirements are such that when the polymer comes into a cavity through a number of gates located at different positions, pressure levels on the surface of a product should be as uniform as possible. Such design can be performed through the intelligent gate positioning to generate the more

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uniform distribution of injection pressure over the product surface.

There have been a number of studies of optimal gate location in the context of CAE filling injection molding design problems where various kinds of optimizer have been employed to conduct design optimization (Kim et al., 1996; Young, 1994; Pandelidis and Zou, 2004; Lin, 2001; Li and Shen, 1995). The paper explores the design of injection mold system using micro genetic algorithm (mGA). Genetic algorithm (conventional GA) is based on the Darwin's theory of the survival of the fittest, and adopts the concept of natural evolution; the competitive designs with more fit are survived by selection, and the new designs are created by crossover and mutation (Lee, 1996; Lee and Hajela, 1996). A conventional GA works with a multiple number of designs in a population. Handling with such designs results in increasing a higher probability of locating a global optimum as well as multiple local optima. GA is also advantageous when the design problem is represented by a mixture of integer/discrete and continuous design variables. Nevertheless, it requires expensive computational costs especially when combining with finite element based CAE analysis tools. A conventional GA determines the population size depending upon the stringlength of a chromosome that is a coded value of a set of design variables. The main difference between a conventional GA and mGA resides on the population size. The population size in mGA is based on Goldberg's concept such that 'Evolution process is possible with small populations to reduce the cost of fitness function evaluation' (Goldberg, 1988). This implies that mGA employs a few number of populations for GA evolution regardless of the number of design variables and the complexity of design parameters (Krishnakumar, 1989; Dennis and Dulikravich, 2001).

The paper discusses the design requirements of filling injection mold optimization to construct the proper objective functions and design constraints. Four different design applications in injection molds are explored to examine the proposed design strategies. The paper shows the effectiveness of mGA in the context of optimization of runner system in injection molding design.

## 2. Mold flow analysis

The flow of a polymer in injection molding process

obeys the following governing equations:

$$\frac{\partial}{\partial x}(S_2 \frac{\partial p}{\partial x}) + \frac{\partial}{\partial y}(S_2 \frac{\partial p}{\partial y}) = 0 \quad (1)$$

$$\rho C_p (\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y}) = \eta \dot{\gamma}^2 + k \frac{\partial^2 T}{\partial z^2} \quad (2)$$

$$\text{where, } S_2 = \int_0^h \frac{z^2}{\eta} dz'$$

In the above equations,  $p$  is a flow pressure,  $T$  is a temperature of polymer, and  $t$  is denoted as time. Parameters  $\eta$ ,  $\gamma$ , and  $k$  are viscosity, shear rate and thermal conductivity, respectively (Lee, 2003). It is assumed that polymer is a non-compaction substance in the filling analysis. The flow analysis in the present study is conducted by Computer Aided Plastics Application (CAPA) (Koo, 2003), a finite element based commercial code for polymer flow of injection molding.

The runner system in injection mold covers the passage of molten polymer from injection port to gates. The present study develops two different runner systems where a cold system requires the change in polymer temperature, and a hot system keep it unchanged while the flow passes through the runner. For the hot runner system has a geometrically consistent thickness due to the constant temperature as shown in Fig. 1a. However, the CAE result of a cold runner system depends on the thickness and shape

Table 1. Ten-bar truss design results.

	micro GA			conventional GA			Reference [20]	
	Case 1	Case 2	Case 3	Case 1	Case 2	Case 3		
Optimal area	X <sub>1</sub>	7.86	8.15	7.85	8.15	7.30	7.81	7.90
	X <sub>2</sub>	0.41	0.18	0.19	0.10	0.83	0.45	0.10
	X <sub>3</sub>	8.38	7.99	8.15	8.20	8.77	8.37	8.10
	X <sub>4</sub>	5.05	3.83	3.89	3.97	3.27	4.16	3.90
	X <sub>5</sub>	0.12	0.96	0.15	1.10	0.75	0.55	0.10
	X <sub>6</sub>	0.41	0.25	0.25	0.10	0.82	0.30	0.10
	X <sub>7</sub>	6.41	5.67	5.87	5.84	6.74	6.30	5.80
	X <sub>8</sub>	5.23	6.29	5.52	5.68	5.06	5.26	5.51
	X <sub>9</sub>	3.83	3.85	5.05	5.07	2.89	3.86	3.68
	X <sub>10</sub>	0.50	0.25	0.25	0.40	1.16	0.42	0.14
Optimal weight	1599	1587	1588	1593	1590	1585	1499	
# of function evaluations	57540	54230	25335	78894	69497	73533		

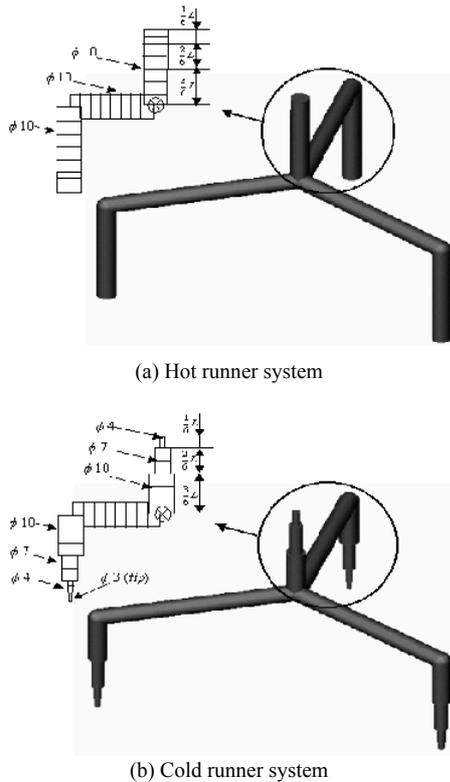


Fig. 1. Modeling of runner system.

shape of a runner. The typical illustration of the geometric model in a cold runner system is shown in Fig.1b where the runner thickness is changed according to the temperature gradient.

### 3. Molding design requirements

#### 3.1 Objective functions

One of the most significant factors considered in the injection molding design is a flow pattern, which implies that a balanced flow should be maintained while a polymer arrives at each part of a design product. Once the improvement on flow balance is obtained, the flow of molten polymer smooths and the maximum injection pressure is decreased with the same or at least evenly distributed injection pressure level at each gate. In a case where the certain part of a product within the mold is filled up earlier than other parts, each part would fall into over-packing and under-packing situations during the filling process of a polymer into mold. Such problem further evokes a malformation like twisting and bending, resulting from the difference in contraction rate during the

course of cooling-off.

The difference in pressure triggers the flow of polymer during the filling process, in which the maximum injection pressure is detected at the injection port of polymer. The polymer always flows from high-pressure region to low-pressure one. When a flow pattern improves, the flow of polymer gets smoother with the maximum injection pressure decreased. However, the flow instability sometimes happens, thereby requiring a higher pressure to fill up. That is, the maximum injection pressure needs to be reduced in order to improve the flow instability. The pressure gap (i.e., the highest and lowest pressure values) among all of gates is also taken as another objective function to determine whether the whole mold is being filled at once.

Most commonly accepted design strategy to improve the flow pattern is the adjustment of gate location. The present study controls the flow pattern by developing the optimal gate positioning problems with proper objective function(s) and design constraints. Objective functions for injection molding design are considered as both ‘maximum injection pressure’ (MIP) and ‘maximum pressure difference’ (MPD). It should be noted that the maximum injection pressure is calculated at the injection port and the maximum pressure difference is a numerical difference between the highest and lowest values of pressure among all of gates. The aforementioned statements could be interpreted as a multiobjective design problem, hence the present study simply employs a weighting method as follows:

$$F(\underline{x}) = \alpha \frac{MIP(\underline{x})}{MIP^*} + \beta \frac{MPD(\underline{x})}{MPD^*} \quad (3)$$

where,  $\alpha$  and  $\beta$  are weighting factors as  $\alpha + \beta = 1$ , and  $\underline{x}$  is a set of design variables which are Cartesian coordinates of gates on a product. Each component in the above equation is normalized by optimal single-objective function value, (i.e.,  $MIP^*$ ,  $MPD^*$ ). It is mentioned that the number of gates is considered as a problem parameter in the study.

#### 3.1 Constraints

Weld-lines are easily detected when more than two flow fronts having different temperature values meet during the filling process. The weld-line is one of the weakest points in molded product; it is very

vulnerable to a shock and subsequently causes external defects of a very glossy polymer. The weld-line should be moved into a less weak region by adjusting the width of a product, the size and/or shape of gates and runners, and the position of gates, etc. The present study considers the position of a weld-line as a constraint in optimal gate positioning of mold design. Once a designer specifies areas where weld-lines should not be generated, all of the finite element nodes in such areas are constrained not to form the weld-lines.

Shear stress is defined as a shear force imposed on the wall of a mold by the shear flow of a polymer. The magnitude of shear stress is proportional to the pressure gradient of each position. In general, the shear stress is zero at the center of a molded product, and reaches a maximum value on the wall. High shear stress triggers the molecule cultivation on the surface of a molded product. Flow instability such as melt fracture has a close relationship with the shear stress. The clear surface of a molded product can be obtained by reducing the magnitude of shear stress. That is, shear stress should be minimized during the mold filling process in order to improve the quality of a molded product, particularly on its surface. Maximum allowable shear stress depends on the kinds of polymer, and is generally taken as 1% of tensile strength of a polymer. Shear stress affecting the quality of end product is considered as another constraint.

### 3.3 Formulation of optimization problem

The statement of a mold design optimization problem can be written as follows:

Find

$$\underline{x}(i, j, k) = \{x_1(i, j, k), x_2(i, j, k), \dots, x_N(i, j, k)\} \quad (4)$$

to minimize

$$F(\underline{x}) = \alpha \frac{MIP(\underline{x})}{MIP^*} + \beta \frac{MPD(\underline{x})}{MPD^*} \quad (5)$$

subject to

$$\text{shear stress}(i, j, k) \leq \text{shear stress allowable} \quad (6)$$

$$\text{weld-line}(i, j, k) = \text{designated area(s) only} \quad (7)$$

$$\text{where, } \underline{x}^{\text{lower}} \leq \underline{x} \leq \underline{x}^{\text{upper}}$$

A set of design variables,  $\underline{x}$  are Cartesian coordinates (i, j, k) of gates on the surface of a molded product, where  $N$  is the number of gates. A traditional weighted-sum method in the context of multiobjective optimization is employed by using two wei-

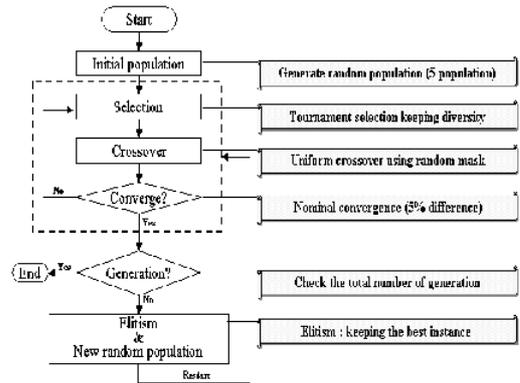


Fig. 2. Micro GA process.

ghting factors of  $\alpha$  and  $\beta$ , where  $\alpha + \beta = 1$ . Multi-objective functions considered in the present study are ‘maximum injection pressure (MIP)’ measured at the injection port and ‘maximum pressure difference (PD)’ among all of gates. The constants, MIP\* and MPD\* are optimal objective function values obtained via single-objective optimization. The permission of weld-lines to designated areas only and the upper limits on shear stress are imposed as design constraints. The flow pattern analysis is performed by CAPA as mentioned in the earlier section, and the optimization is conducted through mGA. It should be noted that Cartesian coordinates (i, j, k) is recognized as nodal points when a molded product is discretized by finite elements in CAPA.

## 4. Micro GA

The overall process of mGA in the present study is depicted in Fig. 2, and a stepwise procedure can be explained as follows:

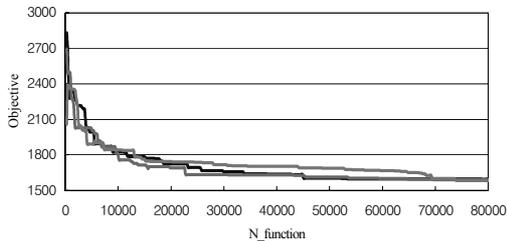
Step-1) Generate an initial population at random. The recommended population size is 3, 5, or 7.

Step-2) Perform a conventional GA evolution until the nominal convergence is satisfied. In the present study, the population size is selected as 5, and a tournament selection operator is used. The crossover probability in mGA is 1.0 due to the small size in population, while a conventional GA is preferred to use it less than 1.0. The nominal convergence means that the difference of 1’s and/or 0’s among string positions is within 5% out of the stringlength, thereby resulting in the convergence to a local solution.

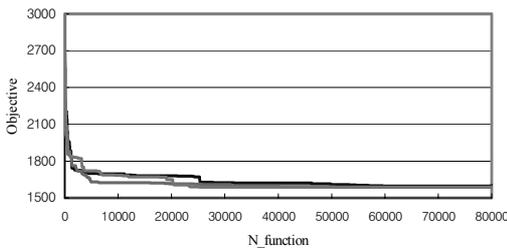
Step-3) During the user-specified number of generations, a new population is updated; one individual is selected by elitism; the remaining individuals in a

new population are generated at random. It should be noted that the selection operation adopts ‘tournament’ for activating the diversity and ‘elitism’ for keeping the best solution. Since the updated populations except for the elitism are generated at random, mGA seldom considers the mutation.

In summary, mGA enables to locate an optimal solution thanks to the small size of populations, tournament and elitism operations in selection, and the full participation in crossover. However, mGA has a drawback upon finding one of multiple local optima only due to the small size of populations and the nominal convergence strategy. A conventional GA is superior to maintaining the diversity while mGA is advantageous of savings in computational resource requirements.



(a) A conventional GA



(b) Micro GA

Fig. 3. Convergence histories of ten-bar truss problem.

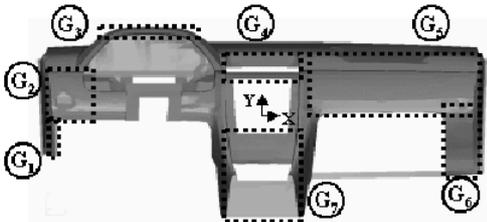


Fig. 4. Seven discrete design spaces for vehicle dashboard problem.

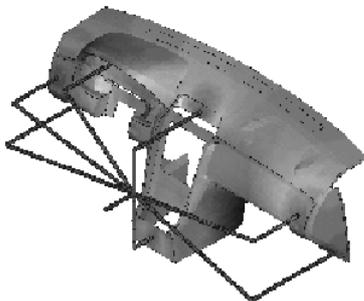


Fig. 5. Initial gate location of vehicle dashboard.

4.1 Truss design

The proposed mGA is verified by a typical ten-bar planar truss optimization problem. The objective is to find optimal cross-sectional areas by minimizing the structural weight subjected to stress constraints (Haftka and Gurdal, 1993). Optimal solutions are obtained via mGA and a conventional GA to compare with each other. The population size in mGA is 5, while a conventional GA requires 250 individuals in a population since the stringlength in this problem is 100. Crossover and mutation probabilities in a conventional GA used are 0.8 and 0.01, respectively. After two genetic search methods are conducted ten times by changing randomly generated initial populations, the most fit design results are demonstrated in Table 1. The convergence history for each optimizer demonstrates that mGA produces the better design and locates the near-optimal solution at the earlier stage of evolution in Fig. 3.

5. Results of design applications

5.1 Vehicle dashboard

A passenger car in-panel has been first examined. This model is supposed to have 7 gates, and design spaces for use in genetic evolution are shown in Fig. 4. Objective functions of MIP and MPD are taken into account, but no constraints are imposed in this model.

The initial design is shown in Fig. 5; this design has been obtained through experience and trial-and-errors in an automotive part molding company. Optimized results by mGA are shown in Figs. 6 to 8, whose objective functions were considered as ‘MIP only’, ‘MPD only’ and ‘both MIP and MPD’, respectively. Design results for each case are summarized in Table 2 as well. It is noted that ‘both MIP and MPD’ is calculated with  $\alpha$  changing from 0.0 to 1.0 with an increment of 0.1 while keeping  $\alpha + \beta = 1.0$ .

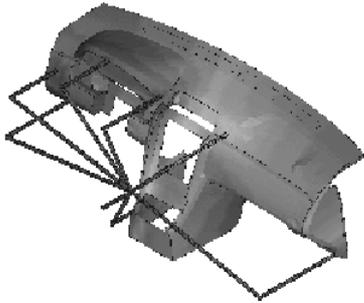


Fig. 6. Optimized design of vehicle dashboard (MIP only).

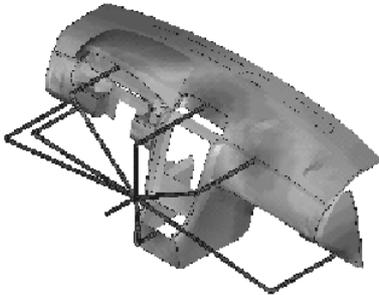


Fig. 7. Optimized design of vehicle dashboard (MPD only).

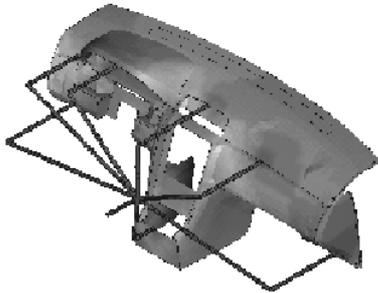


Fig. 8. Optimized design of vehicle dashboard (both MIP and MPD).

In case of ‘MIP only’ in Fig. 6, the maximum injection pressure value has an improvement of 23.9% compared with an initial model, but the pressure distribution on the product becomes worse, resulting in over-packing on the left region. When a case of ‘MPD only’ is considered, the design performance in Fig. 7 is achieved in terms of not only maximum pressure difference but also maximum injection pressure as shown. It is expected that the flow gets smoother during the improvement of pressure distribution, and the maximum injection pressure is decreased as well. In case of ‘both MIP and MPD’ in Fig. 8, its result is quite similar to a case

Table 2. Optimization results of vehicle dashboard.

		maximum pressure [MPa]	maximum difference [MPa]
Initial design		242.69	20.26
objective	MIP only	184.73	35.08
	MPD only	231.22	12.44
	both MIP and MPD	229.92	12.58

Table 3. Optimization results of TV monitor.

		maximum pressure [MPa]	maximum difference [MPa]	shear stress $\leq$ 0.5 [MPa]
Initial design		80.55	13.71	0.45
objective	MIP only	68.46	4.06	0.43
	MPD only	72.27	3.04	0.45
	both MIP and MPD	68.46	4.06	0.43

of ‘MPD only’ in terms of gate locations from Figs. 7 and 8 and the percentile improvement in Table 2. A weighted-sum method is used to obtain the multiobjective optimal solutions by changing  $\alpha$  and  $\beta$  simultaneously, but yields the same results out of a total of 11 weighting factor based trials. The reason why a few number of Pareto solutions are detected is such that the maximum pressure is not counter to pressure distribution in the filling injection molding. In other words, when the overall pressure distribution is improved thanks to the enhancement of flow balance and the smoothness of polymer flow, the maximum pressure is consequently decreased. As far as the pressure distribution of a modeled product is concerned, the change in gate position is noticeable; Gate\_5 of optimized models moves from right to left region compared with an initial model.

### 5.2 TV monitor

The model of a TV monitor equipped with 4 gates is now optimized using objective functions and the upper limit on shear stress constraint, where the shear stress allowable is 0.5MPa. The initial design with 4 discrete design spaces is displayed in Fig. 9, and optimized pressure distributions are shown in Figs. 10 and 11. Design results for single-objective and multiobjective optimization are tabulated in Table 3. In case of ‘MIP only’ generates the same result as weighting method based multiobjective solutions of ‘both MIP and MPD’. In case of ‘MPD only, the maxi-

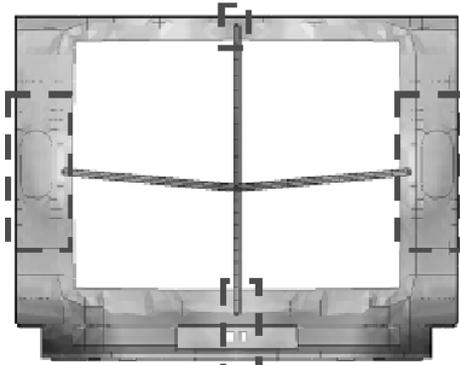


Fig. 9. Initial gate location of TV monitor.

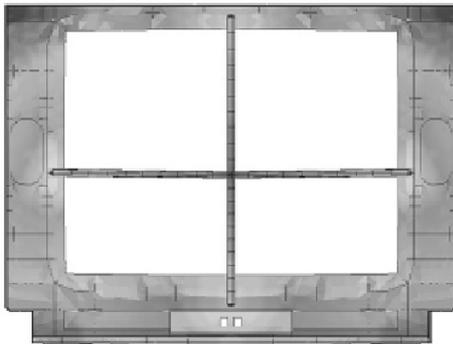


Fig. 10. Optimized design of TV monitor (MPD only).

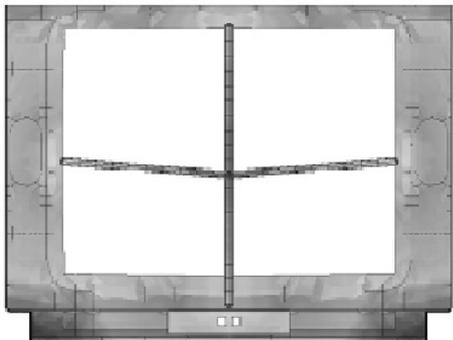


Fig. 11. Optimized design of TV monitor (MIP only & both MIP and MPD).

imum injection pressure and maximum pressure difference have been improved by 10.3% and 77.8%, respectively. It is expected that the enhancement on flow balance and smoothness may be made possible by optimizing the gate positions.

### 5.3 CD tray

The CD tray use in a laptop computer has 4 gates for injection molding. The optimization on this model

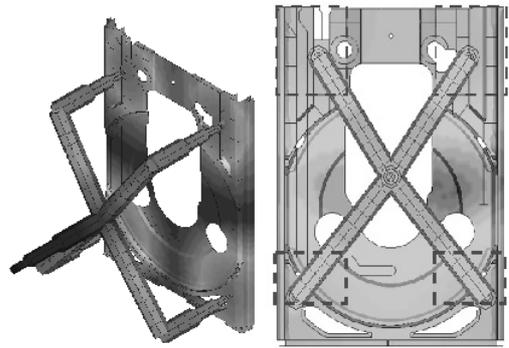


Fig. 12. CD tray (left) and its initial gate location (right).

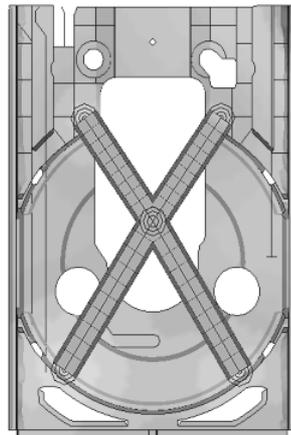


Fig. 13. Optimized design of CD tray (MIP only).

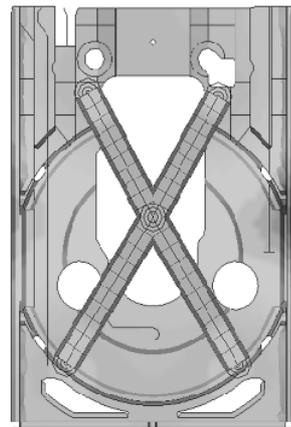


Fig. 14. Optimized design of CD tray (MPD only).

is conducted with a shear stress constraint, where the upper limit on shear stress allowable is 1.5MPa. Initial and optimized results for pressure distribution are shown in Figs. 12 to 15. From the summary of Table 4, the design solutions of optimal objective fun-

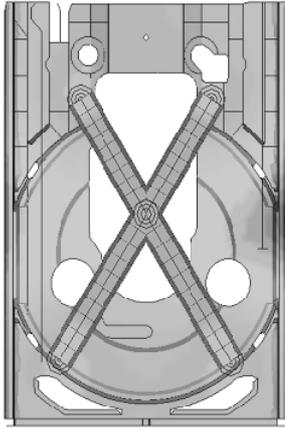


Fig. 15. Optimized design of CD tray (both MIP and MPD).

Table 4. Optimization results of CD tray.

		maximum pressure [MPa]	maximum difference [MPa]	shear stress $\leq$ 1.5 [MPa]
Initial design		82.66	1.192	1.22
objective	MIP only	73.91	7.085	1.26
	MPD only	80.44	0.332	1.12
	both MIP and MPD	78.79	0.376	1.14

ction values in this problem are quite similar to that in the vehicle dashboard. In case of ‘MIP only’, the maximum pressure difference value gets worse than the initial design, even though the maximum injection pressure value has been improved. The cases of ‘MPD only’ and ‘both MIP and MPD’ have turned out that both objective function values are improved. Also, the duplicated multiobjective design solutions are much close to the result obtained by ‘MPD only’, as in the vehicle dashboard design.

#### 5. 4 Plug receptacle

This problem employs the weld-line condition as a constraint instead of shear stress. In Fig. 16, the design space for optimally locating 2 gates is represented by a dotted region, and the restricted areas against weld-lines are designated by 5 solid regions. Actual mold designers do not locate the weld-line restriction just like this problem. Side or rear parts of a product might be preferred. However, this problem places the disjointed 5 weld-line restriction areas in the front to see how much the proposed design strategy of mGA works in the present study. Optimized results of weld-line distribution are shown in Figs. 17 and 18. It is clear to see that all the results are

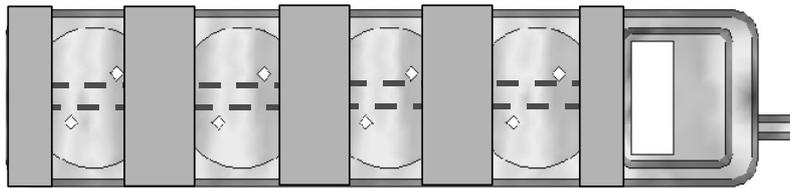


Fig. 16. Design space (dotted area) and weld-line restriction region (solid areas) of plug receptacle.

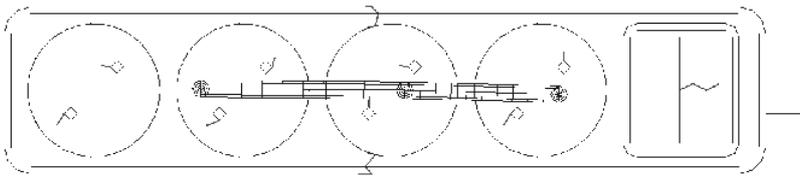


Fig. 17. Weld-line in optimized design of plug receptacle (MPD only).

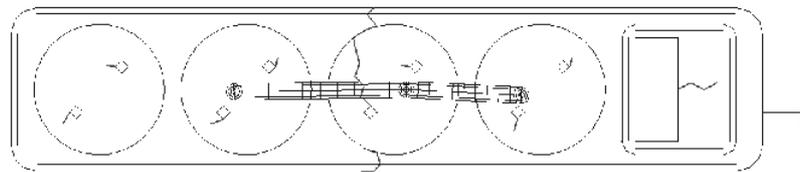


Fig. 18. Weld-line in optimized design of plug receptacle (MIP only & both MIP and MPD).

Table 5. Optimization results of plug receptacle.

		max pressure [MPa]	max difference [MPa]
objective	MIP only	160.30	0.51
	MPD only	166.47	0.05
	both MIP and MPD	160.30	0.51

satisfied with weld-line constraint. The design solutions for optimal objective function values are also similar to those of TV monitor. The solutions of ‘MIP only’ and weighting method based ‘both MIP and MPD’ are the same (see Table 5).

## 6. Concluding remarks

The paper examines micro genetic algorithm in the context of engineering design optimization. Micro genetic algorithm is efficient in handling with small populations over a conventional genetic algorithm. The proposed optimization algorithm is applied to filling injection mold design problem. The central of the paper is to locate gate positions by minimizing both maximum injection pressure at the injection port and maximum pressure difference among all the gates on a product with constraints on shear stress and/or weld-line. Multiobjective design solutions show that the enhancement on flow balance and smoothness may be made possible by optimizing the gate positions. The use of optimized runner systems would subsequently expect to reduce defects such as deformation and twisting that are to be generated during the cooling process.

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