

# Reliability Modeling and Design Optimization for Mechanical Equipment Undergoing Maintenance

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**Abstract**—Design for maintenance is an important design methodology for the life cycle design of electromechanical products or systems. Based on the time-to-failure density function of the part, the reliability model of the mechanical system are developed and the system minimum reliability and steady reliability are defined for maintenance based on reliability simulation during the life cycle of the mechanical system. Secondly, a reliability-based design optimization model for maintenance is presented, in which total life cycle cost is considered as design objective and system reliability as design constrain. Finally, the reliability-based design optimization method for maintenance is illustrated by means of component design demonstrations.

**Keywords**- Maintenance; Reliability; Simulation; Design optimization

## I. INTRODUCTION

During the life cycle of a mechanical product, maintenance is very important to keep the product available and prolong its life. Studies on maintenance for mechanical products are roughly classified into the following three catalogs. (1) How to formulate maintenance policy or (and) how to optimize maintenance periods considering system reliability and maintenance cost<sup>[1-4]</sup>. (2) To develop maintenance methods and tools to ensure system maintenance to both low cost and short repair time, such as special maintenance toolboxes developed<sup>[5-9]</sup>. (3) To design for maintenance, during design procedure, system maintainability is evaluated and is improved<sup>[10-12]</sup>.

Maintenance starts at design. Obviously, design methodology for maintenance, which is one of best effective maintenance means in the life-cycle of a product, attracts many researchers' interests. However, research on design for maintenance is mainly centralized on two fields. One is maintainability evaluation on product design alternatives, the other is some peculiar structures of parts designed for convenient maintenance. In this paper, based on the time-to-failure density function of the part, distributions of service age of parts for a mechanical system that undergoes maintenance are investigated. Then the reliability model of the mechanical system is reconstructed and simulated. Finally, a novel design optimization methodology for maintenance is developed and illustrated by means of design of a link ring for the chain conveyor.

## II. RELIABILITY MODELING OF MECHANICAL SYSTEM FOR MAINTENANCE

### A. Model assumptions

After a mechanical system runs some time, due to replacement of fail parts, primary reliability model is inapplicable to changed system, thus the reliability model should be reconstructed. The mechanical system discussed in this paper has following characteristics: ① System consists of a large number of same type parts, in which the number of parts is constant during the whole life cycle of the system. ② The time-to-failure density distribution functions of all parts are same, also, replacement parts have the same failure distribution functions as the original parts. ③ Failure of each part is a random independent event, i.e., failure of one part does not affect failure of other parts in the system.

### B. Reliability modeling for maintenance

Reliability of a mechanical system depends on its parts, yet reliability and failure probability of which rest on their service ages. Herein, according to the density distribution function of time to failure of the part, part service age distribution of the mechanical system is calculated, then reliability model of the mechanical system for maintenance is developed. During the service of a mechanical system, some parts that fail require to be replaced in time, hence age distribution of parts of the mechanical system undergoing maintenance has been changed. Supposed that after the mechanical system runs some time  $t_n = n\tau$ , where  $\tau$  is time between maintenance activities, i.e., maintenance interval, the unit of  $\tau$  can be hours, days, months, or years. If  $p_i(t_n)$  represents age proportion of parts at  $t_n$  with age  $i\tau$ , thus age distribution of parts at time  $t_n$  denotes matrix  $\{p_0(t_n), p_1(t_n), L, p_i(t_n), L, p_n(t_n)\}$ . The failure density function of parts and current age distribution of parts in the system determine age distribution at next time, or the portion of the contents of each bin that survive to the next time step. An age distribution obtained at each time step for each part population determines failure rate for the following time step. To find failure probability of parts the failure density function is integrated from zero to  $t_n$ . The portion of the population that survives advances to the next age box, and the

portion that fail is replaced by new parts to become zero age to reenter the first box.

Initially, all parts are new and zero age in the first box. That is, at  $t_0 = 0$ , the portion in the first box is

$$p_0(t_0) = 1 \quad (1)$$

At  $t_1 = \tau$ , age fractions of the first box and the second box are represented as

$$\begin{cases} p_1(t_1) = p_0(t_0) \left[ 1 - \int_0^\tau f(x) dx \right] \\ p_0(t_1) = p_0(t_0) \int_0^\tau f(x) dx \end{cases} \quad (2)$$

Portions of both age boxes survive and advance to the next age box, and portions of failed parts from both boxes replaced by new parts appear in the first box.

At  $t_2 = 2\tau$ , the proportions of the first three boxes are calculated as

$$\begin{cases} p_2(t_2) = p_1(t_1) \left[ 1 - \int_0^{2\tau} f(x) dx \right] \\ p_1(t_2) = p_0(t_1) \left[ 1 - \int_0^\tau f(x) dx \right] \\ p_0(t_2) = p_1(t_1) \int_0^{2\tau} f(x) dx + p_0(t_1) \int_0^\tau f(x) dx \end{cases} \quad (3)$$

So, at  $t_n = n\tau$ , portions of parts in each box are calculated by using the following equations.

$$\begin{cases} p_n(t_n) = p_{n-1}(t_{n-1}) \left[ 1 - \int_0^{n\tau} f(x) dx \right] \\ p_{n-1}(t_n) = p_{n-2}(t_{n-1}) \left[ 1 - \int_0^{(n-1)\tau} f(x) dx \right] \\ p_{n-2}(t_n) = p_{n-3}(t_{n-1}) \left[ 1 - \int_0^{(n-2)\tau} f(x) dx \right] \\ \vdots \\ p_2(t_n) = p_1(t_{n-1}) \left[ 1 - \int_0^{2\tau} f(x) dx \right] \\ p_1(t_n) = p_0(t_{n-1}) \left[ 1 - \int_0^\tau f(x) dx \right] \\ p_0(t_n) = \sum_{i=0}^{n-1} p_i(t_{n-1}) \int_0^{(i+1)\tau} f(x) dx \end{cases} \quad (4)$$

Where  $p_0(t_n)$  is the fraction of population of parts with age 0 at  $t_n$ , representing parts that have just been put into service. It means that  $p_0(t_n)$  is failure rate of parts, or replacement rate of failed parts. In other word, the fractions of parts in the first

box at  $t_0, t_1, L, t_n$  are new parts that replace these failed parts.

A series system consists of  $N$  parts that have the same failure density distribution, each part is just a series unit, and each unit is relatively independent. In series system the failure of any one unit results in system failure, in according to the principle of probability multiplication, the reliability of series systems becomes

$$R'(t_n) = \prod_{i=0}^n \left[ 1 - \int_0^{i\tau} f(x) dx \right]^{p_i(t_n)^N} \quad (5)$$

Since the number of parts that comprise the system is constant, here, the system reliability of the mechanical system for maintenance is defined as

$$R(t_n) = \sqrt[N]{R'(t_n)} = \sqrt[N]{\prod_{i=0}^n \left[ 1 - \int_0^{i\tau} f(x) dx \right]^{p_i(t_n)^N}} = \prod_{i=0}^n \left[ 1 - \int_0^{i\tau} f(x) dx \right]^{p_i(t_n)} \quad (6)$$

### III. RELIABILITY SIMULATION FOR MAINTENANCE

Simulation results show that system reliability varies during service. The reliability of a system experiences several oscillations, sometimes is maximum value and then minimum value, finally reaches steady value. Oscillations of system reliability periodically decay, and the period is about the expected life time of parts  $\mu$  (for Weibull distribution, the parameter  $\beta$  approximates expected life at big  $\alpha$ ). For design and maintenance of mechanical systems, minimum value and steady value of system reliability are of importance. Minimum reliability of the system appears at beginning stage, but steady reliability value of the system appears after running a long time. Here, to conveniently discuss later, minimum reliability and steady reliability of the system for maintenance are defined based on simulation results of system reliability shown as in Fig.6.

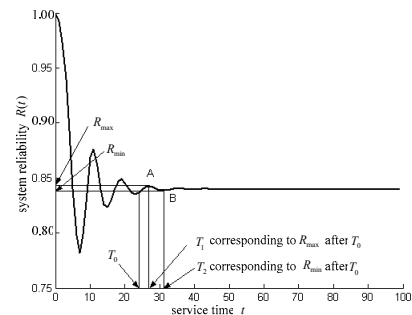


Figure 1. System reliability parameters definition

As it appears at initial phase, minimum reliability of the system can be found in discrete reliability values of simulation

results from  $t=0$  to  $t=2\mu$ . Minimum reliability is defined as

$$R_m = \min(R(t_i)), i=0,1,L,n \quad (7)$$

Supposed that simulation time is  $T_0$ , and  $R_{\max}, R_{\min}$  represent maximum value and minimum value of  $t \in [T_0, T_0 + 2\mu]$  respectively. Once when ratio of maximum reliability value and minimum reliability value  $R_{\max}/R_{\min} > \varepsilon$  is satisfied, system reliability is regarded as arriving at steady value at time  $T_0$ . Thus system reliability, or called as steady reliability, is defined as

$$R_s = (R_{\max} + R_{\min})/2 \quad (8)$$

$\varepsilon \leq 1$  is the stabilization requirement, which could usually be 98%. If  $T_0$  does not exist, system reliability will be unsteady.

#### IV. OPTIMIZATION DESIGN MODEL BASED ON RELIABILITY

A reliability-based design optimization model for maintenance is presented to make a trade-off between the system reliability and life-cycle cost of parts that includes maintenance cost, in which the above models are helpful to calculate part replacement rate of the system, minimum reliability and system reliability. In the model, the cost of life cycle is considered as a design objective, and the reliability of the system is considered as design constraint. The task is to find a design having the minimum cost and satisfying the constraints.

##### A. Model of life cycle cost

Life cycle costs of mechanical systems include production costs and maintenance costs. System maintenance costs are from items listed as follows. (1) cost of parts' replacement, (2) operation cost including cost of resources spent (i.e. labor, equipment) for replacing parts, (3) indirect cost resulting from production interrupt caused by replacing parts, (4) preparation work cost for replacing parts. The foregoing three items are concerned with the number of replacing parts every time of maintenance. The more parts replaced will consume more resource, occupy more production time, thus bring tremendous loss and increase maintenance cost. The last item is not concerned with the number of replacing parts but times of maintenance or replacement. As a result, maintenance costs of mechanical systems are classified as cost considering part replacement number and cost considering maintenance times. In this way, for a mechanical system with a constant number of parts  $N$ , after it runs for time  $M$ , its life cycle cost model, including production cost and maintenance cost, is represent as

$$C = c_0 + \sum_{i=1}^m [c_1 p_0(t_i) + c_2] \quad (9)$$

In Eq. (9),  $C$  is total cost of life cycle of the system for per part in the system.  $c_0, c_1, c_2$  denote coefficient of part production cost, coefficient of replacement cost and

coefficient of preparation cost respectively, and these coefficients can be confirmed by statistical analysis of field datum.  $m = M/\tau$ , where  $M$  represents life of the system. The first term of right-hand side of Eq.(10) represents production cost of the system, the second term of right-hand side of Eq. (9) represents maintenance cost of the system. In Eq. (9),  $c_1 \geq c_0$ , because part replacement cost includes not only production cost of the part that replaces the failed part, but also costs that are spent for resources, and indirect cost caused by replacement. Obviously, the cost that Eq.(10) denotes is not absolute cost, but relative cost. Eq.(9) is also represented as

$$C = c_0 + mc_2 + c_1 \sum_{i=1}^m p_0(t_i) \quad (10)$$

##### B. Model of reliability-based design and optimization

Supposed that a type part of the system has  $n$  design alternatives,  $X = (x_1, x_2, L, x_n)$ , their failure density functions are expressed as  $F = (f_1(t), f_2(t), L, f_n(t))$  corresponding to each alternative.

For a fixed maintenance interval  $\tau^0$ , its reliability-based design optimization model I for maintenance is represented as:

$$\begin{aligned} \min C(x), x \in X \\ \text{s.t. } R_m \geq R_m^0 \\ R_s \geq R_s^0 \end{aligned} \quad (11)$$

Apparently, the minimum life cycle cost and reliability obtained from the above model is responding to the fixed period. For any one of  $n$  design alternatives, its cost and reliability depend on the maintenance interval  $\tau$ . The achievable minimum cost could be obtained from the optimization of the maintenance interval. For the optimal maintenance interval, namely, maintenance interval is optimized to minimized the life cycle cost, thus reliability-based design and optimization model II for maintenance is expressed as

$$\begin{aligned} \min C(x, \tau), x \in X \\ \text{s.t. } R_m \geq R_m^0 \\ R_s \geq R_s^0 \end{aligned} \quad (12)$$

In Eq.(11) and Eq.(12),  $C$  is obtained from Eq.(9) or Eq.(10).  $R_m, R_s$  denotes minimum reliability and steady reliability of the system respectively.  $R_m^0, R_s^0$  is allowable reliability values of the system. In general,  $R_m^0 = (0.75 \sim 0.95)R_s^0$ , which implies system reliability allows to vary in some certain degree during whole life cycle, but variation scope is not over 5%~25% of steady reliability.

### C. Design optimization based on system reliability simulation

Obviously, system steady reliability, minimum reliability and part replacement rate in design models can be derived from reliability simulation. Therefore, design optimization for maintenance is a design methodology based on simulation. In design models, input conditions of reliability simulation are the time to failure density distribution functions of the system part  $F$ , system service life  $M$  and coefficients of life cycle cost are  $c_0, c_1, c_2$ . For maintenance of fixed interval, input conditions add in fixed maintenance interval  $\tau^0$ . Times of maintenance are clearly equal to  $M/\tau^0$  during whole life cycle. As to the situation that maintenance interval needs to be optimized, times of maintenance are rounded  $M/\tau$  to obtain at different maintenance interval. In addition, Design alternatives for the system must satisfy requirement of system reliability, thus  $R_m^0, R_s^0$  are given. Finally, an optimal design alternative and its minimum reliability, steady reliability and life cycle cost are outputted. The flow chart of design optimization for maintenance is shown as Fig.2, in which two models of design optimization for maintenance are integrated. Most possibly, the solution of one model is usually different from another model.

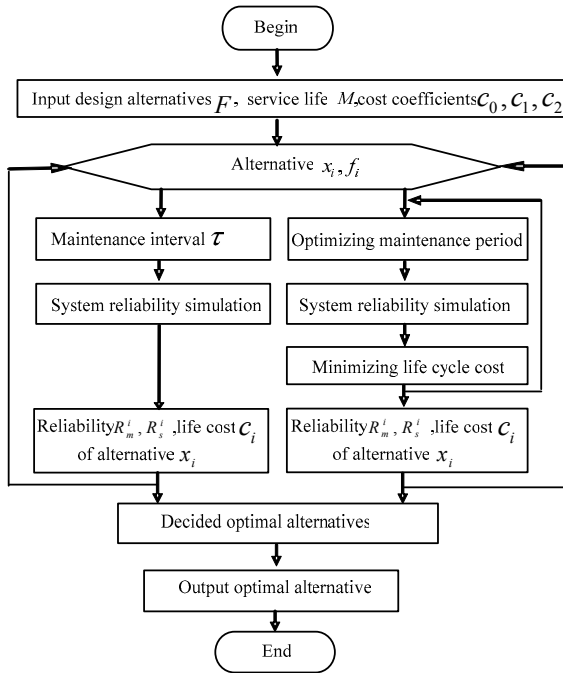


Figure 2. Chart of design optimization for maintenance

## V. DESIGN DEMONSTRATION

There are three design alternatives for link rings of chain conveyors, the service life  $M$  of which is equal to 100 months. The density distribution function of the time to failure of rings is the Weibull distribution, and their distribution parameters and cost coefficients of life cycle are listed in Table 1 as below.

TABLE I. DESIGN ALTERNATIVES AND THEIR PARAMETERS

Scheme $\mathcal{X}$	$\alpha$	$\beta$	$c_0$ /(yuan)	$c_1$ /(yuan)	$c_2$ /(yuan)
1	5	10	3	7	0.5
2	4	12	5	10	0.5
3	4	20	10	18	0.5

Suppose that the requirement of minimum reliability and steady reliability is  $R_s^0 = 0.85, R_m^0 = 0.75$ . Considering that system maintenance interval is selected from a series of equivalent difference values, discrete optimization method is adopted. Simulation results of two design models for maintenance are listed in Table 2. Fig. 8 to Fig. 11 illustrates that system reliability and total life cycle costs vary with service time of the system.

TABLE II. SIMULATION RESULTS OF DESIGN ALTERNATIVES

Scheme $\mathcal{X}$	$\tau^0$ /(month)	$R_m$	$R_s$	$C$ /(RMB)
Identical cycle	1	0.7266	0.8531	131.1384
	2	0.7917	0.8677	157.6288
	3	0.8654	0.9252	180.0370
Scheme $\mathcal{X}$	$\tau^0$ /(month)	$R_m$	$R_s$	$C$ /(RMB)
Optimization cycle	1	0.7734	0.8807	146.8879
	2	0.7917	0.8677	157.6288
	3	0.7820	0.8588	141.5058

Notes:  $\tau^0$  is interval of design model for fixed cycle maintenance Eq. (11), and  $\tau^*$  is optimum interval of design model for optimizing cycle maintenance Eq. (12).

As shown from simulation results listed in Table 2, when system maintenance interval is fixed  $\tau^0 = 1$ , optimum design alternative derived from Eq. (11) is alternative  $x_2$ . Alternative  $x_1$  does not satisfy system reliability constraints, and total life costs of alternative  $x_2$  is lower than that of alternative  $x_3$ . From the example, it could be understood that there could not be a design alternative that would meet system reliability constraints for an inappropriate fixed maintenance interval. When system maintenance interval is optimized, optimum design alternative derived from Eq. (12) is alternative  $x_3$ . In the case, all design alternatives meet the requirements of system reliability, and total life costs of alternative  $x_3$  is the lowest, correspondingly system maintenance interval  $\tau^*$  is 1.8 in the alternative  $x_3$ . It is shown that variable maintenance cycle police leads to different choice of design alternatives, and total life costs can be reduce by optimizing maintenance interval.

Several interesting results could be found from Fig. 3-Fig. 6.

(1) When a fixed interval ( $\tau^0 = 1$ ) is determined, system reliability of alternative  $x_2$  not only satisfies all design requirements but also approaches to requirement value. Reliability of alternative  $x_1$  satisfies the requirement of steady

reliability, but does not satisfy the requirement of minimum reliability in spite of its lowest cost. Though alternative  $x_3$  satisfies the requirement of system reliability, either steady reliability or minimum reliability, it has highest total life cycle costs.

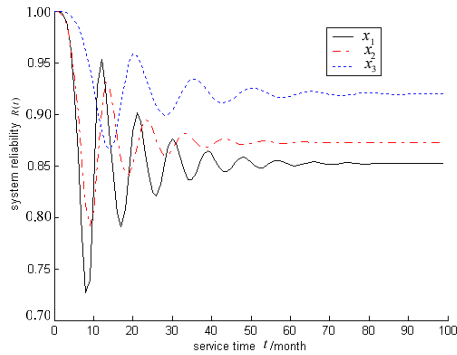


Figure 3. Reliability simulation of design alternatives for a fixed maintenance interval

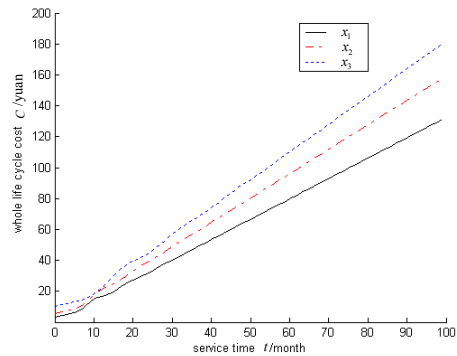


Figure 4. Life cycle costs simulation of design alternatives for a fixed maintenance interval

(2) When maintenance interval is optimized, selection of optimum interval is based on the premise of satisfying requirements of system reliability. As to alternative  $x_1$ , in order to meet requirements of system reliability, maintenance interval decreases,  $\tau^* = 0.8$ , but its total life cost increases somewhat. For alternative  $x_2$ , maintenance interval keeps constant after optimization, also, which means that interval  $\tau = 1$  is an optimum interval for this alternative. For alternative  $x_3$ , due to optimization, maintenance interval increases,  $\tau = 1.8$ , and the difference between system reliability and design requirements reduces, thus it has lower total life cycle costs. Besides, three design alternatives are optimized, their curves of system reliability and total life cycle costs trend to centralization and consistence, and difference of costs among three alternatives reduces.

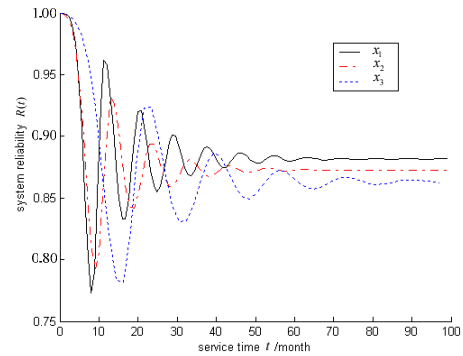


Figure 5. Reliability simulation of design alternatives for an optimum maintenance interval

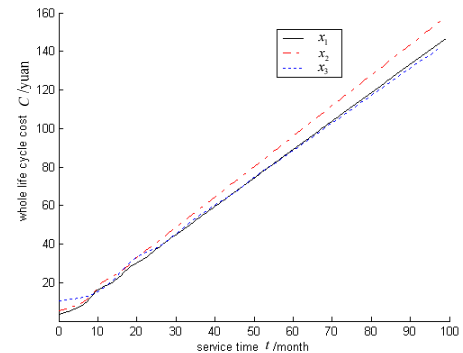


Figure 6. Life cycle costs simulation of design alternatives for an optimum maintenance interval

(3) When the system requires high reliability, correspondingly, maintenance interval will reduce and maintenance costs will rise. On the contrary, when system requires low reliability, correspondingly, maintenance interval will delay, so maintenance costs will reduce, as the decrease of system maintenance costs is subject to system reliability requirement. The steady value and minimum value of system reliability monotonously reduce with the increase of maintenance interval, also, total life cycle costs reduce with the increase of maintenance interval. As a result, minimum interval that steady value and minimum value of system reliability satisfy design requirements will obtain minimum total life cycle costs for the design alternative. It must be pointed out that system reliability of the design alternative is not equal to but little more than the requirement values due to adoption of discrete optimization.

(4) When design alternatives of the system are decided, the optimum choice of design alternatives depends on not only maintenance interval but also requirement of system reliability and system service life. For example, when interval is fixed ( $\tau^0 = 1$ ), and system reliability required reduces from  $R_m^0 = 0.75$  to  $R_m^0 = 0.70$ , the optimum design alternative derived from Eq.(11) is alternative  $x_1$  instead of alternative  $x_2$ . When system service life switches from  $M = 100$  to 50, the optimum design alternative obtained from Eq. (12) is alternative  $x_1$  replacing alternative  $x_3$  shown as Fig. 6. It

means that, since parts made by high quality materials have long service life, the design alternative obtains lower total life cycle costs in spite of their higher production costs.

## VI. CONCLUSION

Maintenance is one of critical tasks during life cycle of the product. Replacement of parts will cause the change of system reliability and life cycle costs. Based on the time-to-failure density function of parts, steady reliability, minimum reliability and life cycle costs can be obtained by means of reconstruction of reliability model and simulation of system reliability. This paper develops reliability-based design optimization methodology for maintenance, in which total life cycle costs are regarded as the design object and system reliability as design constraints. It provides a new approach to make a trade-off between the reliability and total life cycle costs of the mechanical system in design optimization for maintenance.

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