

# NOVEL METHOD OF REALIZING THE OPTIMAL TRANSMISSION OF THE CRANK-AND-ROCKER MECHANISM DESIGN

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**Abstract:** A novel method of realizing the optimal transmission of the crank-and-rocker mechanism is presented. The optimal combination design is made by finding the related optimal transmission parameters. The diagram of the optimal transmission is drawn. In the diagram, the relation among minimum transmission angle, the coefficient of travel speed variation, the oscillating angle of the rocker and the length of the bars is shown, concisely, conveniently and directly. The method possesses the main characteristic. That it is to achieve the optimal transmission parameters under the transmission angle by directly choosing in the diagram, according to the given requirements. The characteristics of the mechanical transmission can be improved to gain the optimal transmission effect by the method. Especially, the method is simple and convenient in practical use.

**Key words:** Crank-and-rocker mechanism Optimal transmission angle  
Coefficient of travel speed variation

## 0 INTRODUCTION

By conventional method of the crank-and-rocker design, it is very difficult to realize the optimal combination between the various parameters for optimal transmission. The figure-table design method introduced in this paper can help achieve this goal. With given conditions, we can, by only consulting the designing figures and tables, get the relations between every parameter and another of the designed crank-and-rocker mechanism. Thus the optimal transmission can be realized.

The concerned designing theory and method, as well as the real cases of its application will be introduced later respectively.

## 1 ESTABLISHMENT OF DIAGRAM FOR OPTIMAL TRANSMISSION DESIGN

It is always one of the most important indexes that designers pursue to improve the efficiency and property of the transmission. The crank-and-rocker mechanism is widely used in the mechanical transmission. How to improve work ability and reduce unnecessary power losses is directly related to the coefficient of travel speed variation, the oscillating angle of the rocker and the ratio of the crank and rocker. The reasonable combination of these parameters takes an important effect on the efficiency and property of the mechanism, which mainly indicates in the evaluation of the minimum transmission angle.

The aim realizing the optimal transmission of the mechanism is how to find the maximum of the minimum transmission angle. The design parameters are reasonably combined by the method of lessening constraints gradually and optimizing separately. Consequently, the complete constraint field realizing the optimal transmission is established.

The following steps are taken in the usual design method. Firstly, the initial values of the length of rocker  $l_3$  and the oscillating angle of rocker  $\varphi$  are given. Then the value of the coefficient of travel speed variation  $K$  is chosen in the permitted range. Meanwhile, the coordinate of the fixed hinge of crank  $A$  possibly realized is calculated corresponding to value  $K$ .

### 1.1 Length of bars of crank and rocker mechanism

As shown in Fig.1, left arc  $C_2G$  is the permitted field of point

$A$ . The coordinates of point  $A$  are chosen by small step from point  $C_2$  to point  $G$ .

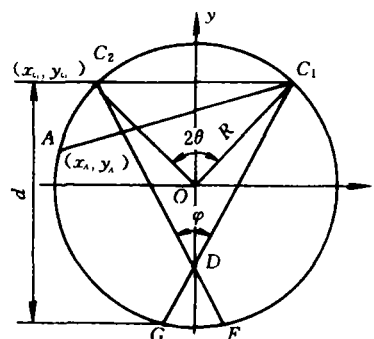


Fig.1 Position of point A

The coordinates of point  $A$  are

$$y_A = y_{C_2} - h_0 \quad (1)$$

$$x_A = \sqrt{R^2 - y_A^2} \quad (2)$$

where  $h_0$ , the step, is increased by small increment within range  $(0, H)$ . If the smaller the chosen step is, the higher the computational precision will be.  $R$  is the radius of the design circle.  $d$  is the distance from  $C_2$  to  $G$ .

$$d = l_3 \cos \frac{\varphi}{2} + \left[ 2R \cos \left( \frac{\varphi}{2} - \theta \right) - l_3 \right] \cos \frac{\varphi}{2} \quad (3)$$

Calculating the length of arc  $AC_1$  and  $AC_2$ , the length of the bars of the mechanism corresponding to point  $A$  is obtained<sup>[1,2]</sup>.

### 1.2 Minimum transmission angle $\gamma_{\min}$

Minimum transmission angle  $\gamma_{\min}$  (see Fig.2) is determined by the equations<sup>[3]</sup>

$$\cos \gamma_{\min} = \frac{l_2^2 + l_3^2 - (l_4 - l_1)^2}{2l_2l_3} \quad (4)$$

$$\cos \gamma_{\max} = \frac{l_2^2 + l_3^2 - (l_4 + l_1)^2}{2l_2l_3} \quad (5)$$

$$\gamma'_{\min} = 180^\circ - \gamma_{\max} \quad (6)$$

where  $l_1$ —Length of crank(mm)  
 $l_2$ —Length of connecting bar(mm)  
 $l_3$ —Length of rocker(mm)  
 $l_4$ —Length of machine frame(mm)

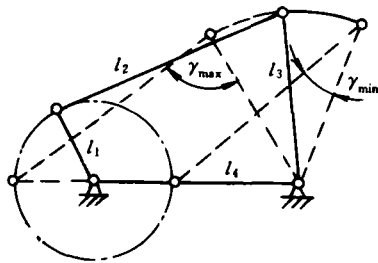


Fig.2 Position of minimum angle of transmission

Firstly, we choose minimum comparing  $\gamma_{min}$  with  $\gamma'_{min}$ . And then we record all values of  $\gamma_{min}$  greater than or equal to  $40^\circ$  and choose the maximum of them.

Secondly, we find the maximum of  $\gamma_{min}$  corresponding to any oscillating angle  $\phi$  which is chosen by small step in the permitted range (maximum of  $\gamma_{min}$  is different with the coefficient of travel speed variation  $K$ ).

Finally, we change the length of rocker  $l_3$  by small step similarly. Thus we may obtain the maximum of  $\gamma_{min}$  corresponding to the different length of bars, different oscillating angle  $\phi$  and the coefficient of travel speed variation  $K$ .

Fig.3 is accomplished from Table for the purpose of diagram design.

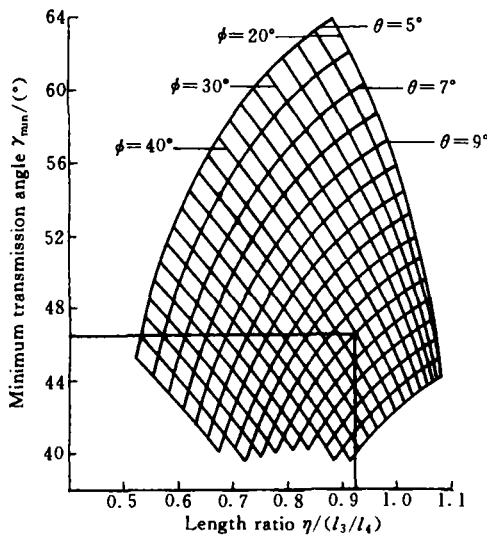


Fig.3 Figure of the optimal design

Table Part of computational value

K=1.117 ( $\theta=10^\circ$ )					
$\phi(^{\circ})$	$\gamma_{min}(^{\circ})$	$l_1/mm$	$l_2/mm$	$l_3/mm$	$l_4/mm$
20	55.994 1	16.650 2	58.968 0	100.00	99.954 2
30	54.171 9	24.991 7	81.157 3	100.00	109.166 0
40	51.411 2	33.135 5	102.723 5	100.00	121.100 4
50	48.364 8	41.108 7	119.772 7	100.00	132.926 1
60	44.860 4	48.759 7	136.022 4	100.00	144.744 3
70	41.072 4	56.119 4	147.128 3	100.00	153.398 9

K=1.182 ( $\theta=15^\circ$ )					
$\phi(^{\circ})$	$\gamma_{min}(^{\circ})$	$l_1/mm$	$l_2/mm$	$l_3/mm$	$l_4/mm$
20	49.607 2	16.417 4	46.347 8	100.00	94.742 4
30	48.615 8	24.390 8	70.674 7	100.00	99.558 8
40	46.539 3	32.449 5	89.018 7	100.00	107.797 3
50	44.092 4	40.377 6	103.799 6	100.00	116.955 4
60	41.071 4	47.981 6	117.871 3	100.00	126.222 6
70	<40				

It is worth pointing out that whatever the length of rocker  $l_3$  is evaluated, the location that the maximum of  $\gamma_{min}$  arises is only related to the ratio of the length of rocker and the length of machine frame  $l_3/l_4$ , while independent of  $l_3$ .

## 2 DESIGN METHOD

### 2.1 Realizing the optimal transmission design given the coefficient of travel speed variation and the maximum oscillating angle of the rocker

The design procedure is as follows.

(1) According to given  $K$  and  $\phi$ , taken account to the formula

$$\theta = \frac{K-1}{K+1} \times 180^\circ \quad (7)$$

the extreme included angle  $\theta$  is found. The corresponding ratio of the length of bars  $l_3/l_4$  is obtained consulting Fig.3.

(2) Chosen the length of rocker  $l_3$  according to the work requirement, the length of the machine frame is obtained from the ratio  $l_3/l_4$ .

(3) Choose the centre of fixed hinge  $D$  as the vertex arbitrarily, and plot an isosceles triangle, the side of which is equal to the length of rocker  $l_3$  (see Fig.4), and  $\angle C_1DC_2 = \phi$ . Then plot  $C_2M \perp C_1C_2$ , draw  $C_1N$ , and make angle  $\angle C_2C_1N = 90^\circ - \theta$ . Thus the point of intersection of  $C_2M$  and  $C_1N$  is gained. Finally, draw the circum-circle of triangle  $\triangle PC_1C_2$ .

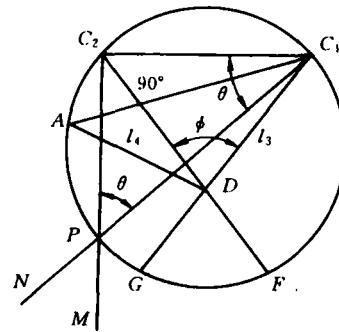


Fig.4 Case illustration

(4) Plot an arc with point  $D$  as the centre of the circle,  $l_4$  as the radius. The arc intersections arc  $C_2G$  at point  $A$ . Point  $A$  is just the centre of the fixed hinge of the crank.

Therefore, from the length of the crank

$$l_1 = (AC_1 - AC_2)/2 \quad (8)$$

and the length of the connecting bar

$$l_2 = AC_1 - l_1 \quad (9)$$

we will obtain the crank and rocker mechanism consisted of  $l_1, l_2, l_3$ , and  $l_4$ . Thus the optimal transmission property is realized under given conditions.

### 2.2 Realizing the optimal transmission design given the length of the rocker (or the length of the machine frame) and the coefficient of travel speed variation

We take the following steps.

(1) The appropriate ratio of the bars  $l_3/l_4$  can be chosen according to given  $K$ . Furthermore, we find the length of machine frame  $l_4$  (the length of rocker  $l_3$ ).

(2) The corresponding oscillating angle of the rocker can be obtained consulting Fig.3. And we calculate the extreme included angle  $\theta$ .

Then repeat (3) and (4) in section 2.1.

## 3 DESIGN EXAMPLE

The known conditions are that the coefficient of travel speed variation  $K=1.1818$  and maximum oscillating angle  $\phi=40^\circ$ . The crankandrocker mechanism realizing the optimal transmission is designed by the diagram solution method presented above.

First, with Eq.(7), we can calculate the extreme included an-

gle  $\theta=15^\circ$ . Then, we find  $l_3/l_4=0.93$  consulting Fig.3 according to the values of  $\theta$  and  $\varphi$ .

If evaluate  $l_3=50$  mm, then we will obtain  $l_4=50/0.93=53.76$  mm.

Next, draw sketch(omitted).

As result, the length of bars is  $l_1=16$  mm,  $l_2=46$  mm,  $l_3=50$  mm,  $l_4=53.76$  mm.

The minimum transmission angle is

$$\gamma_{\min} = \arccos \frac{l_2^2 + l_3^2 - (l_4 - l_1)^2}{2l_2l_3} = 46.3698^\circ$$

The results obtained by computer are  $l_1=16.2227$  mm,  $l_2=44.5093$  mm,  $l_3=50.0000$  mm,  $l_4=53.8986$  mm.

Provided that the figure design is carried under the condition of the Auto CAD circumstances, very precise design results can be achieved.

#### 4 CONCLUSIONS

A novel approach of diagram solution can realize the optimal

transmission of the crank-and-rocker mechanism. The method is simple and convenient in the practical use. In conventional design of mechanism, taking 0.1 mm as the value of effective the precision of the component sizes will be enough.

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