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Crack initiation at contact surface

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Abstract

By using the S-theory, the crack initiation angle from the contact surface of rectangular rigid punch and flat-surface substrate has been investigated. The coefficient of friction at the edge of contact, which characterizes the asymptotic stress field, is considered as the controlling parameter in the analysis. The predicted results are in agreement with the experimental observations. The information gained may lend insight into the different stages of damage associated with the complex process of fretting fatigue.

Keywords: Fracture; Fretting fatigue; Mechanics; Crack nucleation

1. Introduction

Contact problem covers a wide range of engineering problems. They may include splint joints between shafts, bolted connections, and certain geometries of gas turbine fan blade dovetails etc. These components are prone to failure by fatigue. Fretting fatigue failure can be divided into two stages: crack initiation and propagation. Life estimate of cracked structures may be solved by many existing models. But the effect of fretting contact on crack initiation is likely to be far less amenable to analysis because crack nucleation and initiation are more complex as the problem may involve a knowledge of the combined interaction of load, geometry and material in addition to the use of a valid failure criterion that preferably could handle crack nucleation, initiation and propagation.

Fatigue life and endurance limit of mechanical components are reduced by cyclic fretting fatigue involving contact. Cyclic fretting loads tend to activate flaw at the contact surface which will lead to the development of cracks. However, the analysis of fretting fatigue is difficult because the damage process may involve a multitude of cracks [1,2]. In many test, multiple cracks are often found near the edge of contact or near the ship-stick boundary [3,4]. Conventional approach does not consider the presence of cracks and makes use of stress and strain in certain planes along the contact surface that are regarded as critical [5–8]. The stress and strain, however, are highly elevated near the contact edge similar to that of a crack tip. A slight change in the fretting conditions, can cause large changes in the final fracture. This would lead to large scatter in the data. The present work will

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use contact and fracture mechanics for predicting the direction of crack initiation from the contact surface into the contacting body.

2. Strain energy density criterion

The S-criterion can be used to describe the fracture behavior of stable and unstable crack growth, critical loads and crack initiation angle in mixed mode. It makes use of the strain–energy–density factor S which is related to the strain energy density function $\mathrm{d}W/\mathrm{d}V$ by the relation $\mathrm{d}W/\mathrm{d}V = S/r$ where r is the distance from the crack tip. The critical value of $\mathrm{d}W/\mathrm{d}V$ can be determined from the area under the uniaxial stress and strain curve [9–11]. For the asymptotic analysis S depends only on the angle θ. In general, S can depend on r. In what follows, it suffices to use the asymptotic form of S:

$$S = a_{11}K_{\mathrm{I}}^2 + a_{12}K_{\mathrm{I}}K_{\mathrm{II}} + a_{22}K_{\mathrm{II}}^2 + a_{33}K_{\mathrm{III}}^2 \tag{1}$$

where

$$\begin{aligned} a_{11} &= \frac{1+\mu}{8\pi E}(3 - 4\mu - \cos\theta)(1 + \cos\theta) \\ a_{12} &= \frac{1+\mu}{8\pi E}(2\sin\theta)(\cos\theta - 1 + 2\mu) \\ a_{22} &= \frac{1+\mu}{8\pi E}[4(1-\mu)(1-\cos\theta) \\ &\quad + (1+\cos\theta)(3\cos\theta - 1)] \\ a_{33} &= \frac{1+\mu}{2\pi E} \end{aligned} \tag{2}$$

with μ being the Poisson's ratio and E the elastic modulus.

For the past several decades, S-theory has been widely used in analysis of crack problems by using the asymptotic stress field on $r^{-1/2}$. However, the theory remains valid when using the complete stress field. In this case, S would depend on r and θ. It should be pointed out that the relation $\mathrm{d}W/\mathrm{d}V = S/r$ is necessitated from introducing a distance from the crack tip to locate the site of failure. This distance is r which coincides with the singular character of the strain energy density field. Moreover, r can also be used as the distance from the contact edge in the absence of a pre-existing crack. Hence, it can be used to predict the direction of crack initiating from the contact.

3. Asymptotic stress field in sliding contact

3.1. Boundary condition

The sliding contact of a rigid, square-ended punch, of half width a, on a homogeneous, isotropic, elastic body in half plane is shown in Fig. 1. The Cartesian coordinates (x_1, x_2), and the polar coordinates (r, θ), both with the origin at left edge of contact, are selected. Normal force P and tangential force Q act on the punch. The normal and shear tractions along interface have been solved in closed form [2,12], which are

$$p(x_1) = -\frac{P\sin\lambda\pi}{\pi}\left(2 - \frac{x_1}{a}\right)^{\lambda-1}\left(\frac{x_1}{a}\right)^{-\lambda} \tag{3}$$

and

$$q(x_1) = fp(x_1) \tag{4}$$

where f is the coefficient of friction; the λ can be determined by

$$\tan\lambda\pi = \frac{2(1-\mu)}{f(1-2\mu)}, \quad 0 < \lambda < 1 \tag{5}$$

with μ being the Poisson's ratio of the substrate. Eq. (3) shows that the stress state near the punch corners varies as

$$\sigma_{ij} = \begin{cases} 0(r^{\lambda-1}), & x_1 = 2a \\ 0(r^{-\lambda}), & x_1 = 0 \end{cases} \quad \text{as } r \to 0 \tag{6}$$

When $\mu = 0.5$ or with $f = 0$, it yields $\lambda = 0.5$. This gives the same order of stress singularity as

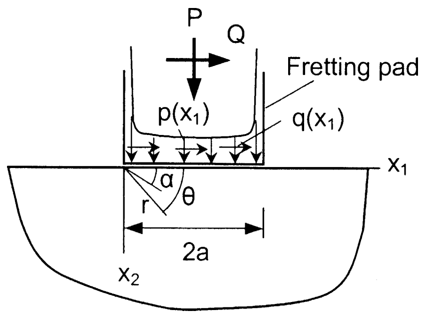


Fig. 1. Contact between a two-dimensional rectangular punch and a substrate.

that for a crack from Eqs. (5) and (6). However, the stress intensity factor is not related to the energy release rate as in the crack problem. Therefore, the physical meaning of K_I when used as a failure criterion is not clear. The strain energy density theory does not have this limitation since it focuses attention on the failure of an element referenced from the vicinity of a site where failure is likely to occur. In passing, it may be emphasized that failure is always assumed to initiate from a finite distance away from the crack tip whose exact location can never be found in the material.

When $\mu = 0.5$, the asymptotic stress boundary conditions of substrate in contact area next to left corner are

$$\sigma_{22}|_{\theta=0} = -\frac{P}{\pi\sqrt{2ar}} \tag{7}$$

and

$$\sigma_{21}|_{\theta=0} = -\frac{fP}{\pi\sqrt{2ar}} \tag{8}$$

3.2. Mode I singular stress field

The singular stress fields at the sharp edge of the contact between the rectangular rigid punch and substrate are known from the asymptotic contact analyses [12]. Using the polar coordinates (r, θ), Fig. 1, the stresses at the left corner are found to vary as

$$\begin{pmatrix} \sigma_{rr}^{I} \\ \sigma_{\theta\theta}^{I} \\ \sigma_{r\theta}^{I} \end{pmatrix} = -\frac{K_I}{\sqrt{2\pi r}} \begin{pmatrix} \cos\frac{\theta}{2}\left(1 + \sin^2\frac{\theta}{2}\right) \\ \cos^3\frac{\theta}{2} \\ \sin\frac{\theta}{2}\cos^2\frac{\theta}{2} \end{pmatrix} \tag{9}$$

This expression is identical in form with the Mode I compressed stress field, but where

$$K_I = \frac{P}{\sqrt{\pi a}} \tag{10}$$

3.3. Mode II singular stress field

From the solution in [12], the asymptotic stress field of Mode II is given by

$$\begin{pmatrix} \sigma_{rr}^{II} \\ \sigma_{\theta\theta}^{II} \\ \sigma_{r\theta}^{II} \end{pmatrix} = -\frac{K_{II}}{\sqrt{2\pi r}} \begin{pmatrix} \sin\frac{\theta}{2}\left(1 - 3\sin^2\frac{\theta}{2}\right) \\ -3\sin\frac{\theta}{2}\cos^2\frac{\theta}{2} \\ \cos\frac{\theta}{2}\left(1 - 3\sin^2\frac{\theta}{2}\right) \end{pmatrix} \tag{11}$$

where

$$K_{II} = \frac{fP}{\sqrt{\pi a}} \tag{12}$$

3.4. Characters of the stress fields

The initiation of a crack depends on singular stress fields. This stress fields is equal to the Eq. (9) adding Eq. (11). Then the local (effective) k_I and k_{II}, are expressed in terms of θ as [13]

$$k_I(\theta) = \sigma_{\theta\theta}(\theta)\sqrt{2\pi r} \tag{13}$$

$$k_{II}(\theta) = \sigma_{r\theta}(\theta)\sqrt{2\pi r} \tag{14}$$

where $\sigma_{\theta\theta}(\theta) = \sigma_{\theta\theta}^{I} + \sigma_{\theta\theta}^{II}$ and $\sigma_{r\theta}(\theta) = \sigma_{r\theta}^{I} + \sigma_{r\theta}^{II}$. Fig. 2 shows the typical angular variations of Eqs. (13) and (14) with angle θ and f. It indicates that $k_I(\theta)$ will be positive at certain θ and f, which means that the contact boundary possesses typical fracture characteristics that can be investigated by using the S-theory.

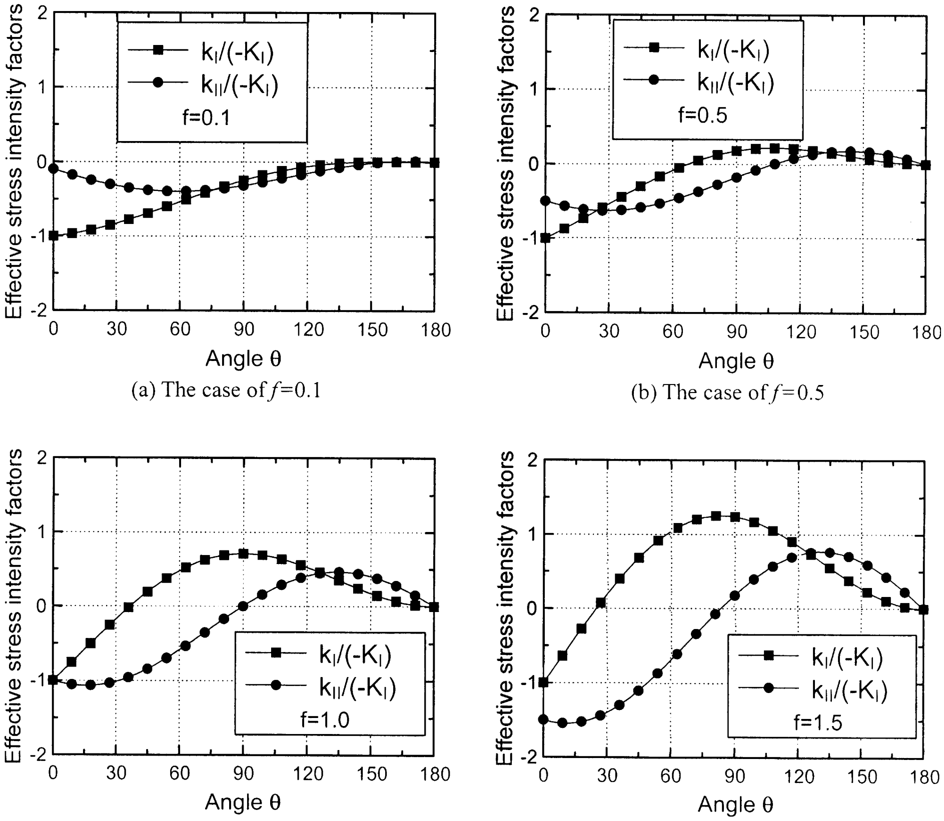
4. Boundary cracking in complete fretting contact

For singular stress fields next to the corners, a new crack will form at a certain critical load. The crack initiation angles can be obtained from the strain energy density factor criterion [9–11]. It is assumed that the crack tends to run in the θ_0 direction for which S_{min} prevails. That is $\partial S/\partial\theta_0 = 0$. This leads to

$$a'_{11} + a'_{12}f + a'_{22}f^2 = 0 \tag{15}$$

where

$$\begin{aligned} a'_{11} &= \sin\theta_0(2\mu - 1 + \cos\theta_0) \\ a'_{12} &= 2\cos^2\theta_0 - (1 - 2\mu)\cos\theta_0 - 1 \\ a'_{22} &= \sin\theta_0(1 - 2\mu - 3\cos\theta_0) \end{aligned} \tag{16}$$



(a) The case of f=0.1
(b) The case of f=0.5
(c) The case of f=1.0
(d) The case of f=1.5

Fig. 2. Variation of effective stress intensity factors k_I, k_{II} with angle θ, for different f.

Fig. 3 gives schematically the relationship of normalized cracking angle θ_0/π and the coefficient of friction. Experimental observations [13] indicate that the crack initiation angle in fretting fatigue tests ranges widely from 25° to 80°. In these fretting tests, f varies with cycling from initial value 0.2–0.4 to final stable value 0.7–1.2. Fig. 3 indicates that the cracking angles predicted by S-theory for crack initiating from the contact boundary are in agreement with the experimental results.

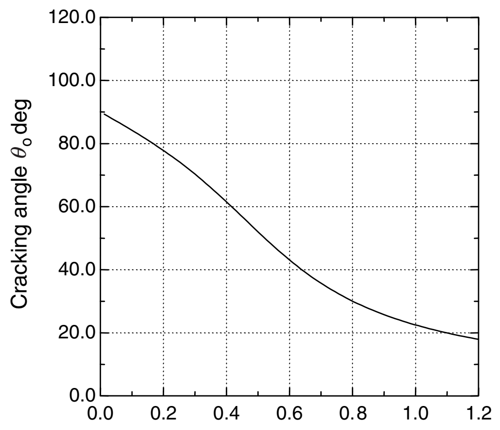


Fig. 3. Cracking angle θ_0 versus friction coefficient f.

5. Conclusions

The problem of contact between a rectangular rigid punch and flat-surface substrate has been investigated by using the S-theory. Cracking angles in boundary of the substrate are theoretically predicted. The present findings provide a method for determining crack initiation from a site that had no existing crack. Keep in mind that a pre-requisite for using fracture mechanics is that the material must be assumed to have an existing crack.

Additionally, the work in [14] studied the stress state near the corner of complete contact subject to fretting action using an asymptotic analysis. Their results indicated that there are infinite combinations of friction coefficient f and wedge angle φ that would result for a -0.5 singularity next to the corner. Even though the order of the stress singularity from the contact boundary without a crack is the same as that for a re-existing crack, the physical interpretation of the stress intensity factors for the contact problem is different, the meaning of which is not clear. The use of the strain energy density theory does not have this problem. Its physical meaning in terms of predicting failure by fracture remains the same. This distinction should be recognized.

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