



# Optimal Design of Compliant Trailing Edge for Shape Changing

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## Abstract

Adaptive wings have long used smooth morphing technique of compliant leading and trailing edge to improve their aerodynamic characteristics. This paper introduces a systematic approach to design compliant structures to carry out required shape changes under distributed pressure loads. In order to minimize the deviation of the deformed shape from the target shape, this method uses MATLAB and ANSYS to optimize the distributed compliant mechanisms by way of the ground approach and genetic algorithm (GA) to remove the elements possessive of very low stresses. In the optimization process, many factors should be considered such as airloads, input displacements, and geometric nonlinearities. Direct search method is used to locally optimize the dimension and input displacement after the GA optimization. The resultant structure could make its shape change from 0 to 9.3 degrees. The experimental data of the model confirms the feasibility of this approach.

*Keywords:* adaptive wing; compliant mechanism; genetic algorithm; topology optimization; distributed pressure load; geometric nonlinearity

## 1 Introduction

As conventional airfoil contours are usually designed with specific lift coefficients and Mach numbers, they could not change in accordance with the environment changing. Siclari and Austin<sup>[1]</sup> indicated that the variable camber trailing edge would produce the drag about sixty percent less than the conventional fixed camber airfoil.

There are three methods used to design variable camber wings. Of them, one is conventional hinged mechanism, which, however, will create discontinuities over the wing's surface leading to earlier airflow separation and drag increase. The others are smart material and the compliant mechanism, of which both could realize smooth shape

changing. Nevertheless, compared to the compliant mechanism, the smart-material-made actuators have many disadvantages, such as deficient in energy, slow in response, strong in hysteresis, limited by temperature, and difficult to control too many actuators. Musolf<sup>[2]</sup> from Industry University of Berlin used Ni-Ti shape-memory-alloy wire to make an adaptive variable camber wing, which could quickly change its shape, but could not perform highly frequent alteration because of its resilience dependent on the heat exchange with the outside environment.

Compliant mechanism is a kind of one-piece flexible structure, which can transfer motion and power through its own elastic deformation. It is not only flexible enough to deform, but also has enough stiffness to withstand external loads. Thanks to its joint-free nature, it does not have the troublesome problems confronted by conventional mechanism,

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such as friction, lubrication, noise and recoiling, thereby achieving smooth shape changing.

In 1994, Kota<sup>[3]</sup>, a professor from University of Michigan, firstly pointed out that compliant mechanism could be used to control static shape changing under the sponsorship of the Air Force Office of Scientific Research in USA. Saggere and Kota<sup>[4]</sup> suggested a new method to design compliant adaptive structures, which made the least square errors between the shape-changed curve and the target curve as the objective function for optimization. Based on their work, Lu<sup>[5]</sup> put forward a load path representation method. However, her work was limited to only linear analysis under consideration of nodal loads. Good<sup>[6]</sup> from Virginia Polytechnic Institute of State University used the compliant mechanism and the Moving Asymptotes method to design the fuselage tail within the allowable range of its tip maximal deflection. Kota and Hetrick<sup>[7]</sup> in 2004 designed a compliant trailing edge on the base of the F16's data, which can change from 0° to 15° and obtained a patent. Campanile<sup>[8]</sup> from German Aerospace Center presented a modal procedure to design synthetic flexible mechanisms for airfoil shape control, and pointed out that the future research should take into account the airload and the geometric nonlinearity. Buhl<sup>[9]</sup> from Risø National Laboratory of the Wind Energy Department in Denmark used the SIMP method and geometrically nonlinear finite element method to design compliant trailing edge flaps. FlxSys Inc in 2006 produced an adaptive compliant wing, which stood the test on the White Knight airplane. The results indicated that the compliant trailing edge could change  $\pm 10^\circ$ <sup>[10]</sup>. In China, the research of adaptive wing has been concentrated on smart material and conventional mechanism. Few people, it seems, have worked on designing adaptive wings with the compliant mechanism. Yang is an exception. He analyzed the active aero-elastic wings based on the aero-servo-elasticity technology<sup>[11]</sup>. Chen and Huang separately investigated the morphing of the compliant leading edge from the viewpoints of discreteness and continuity<sup>[12-13]</sup>.

This paper presents a method to design the shape changeable structure by MATLAB and ANSYS associated with distributed compliant mechanism on the base of the ground structure approach and genetic algorithm (GA) taking into account the external distributed loads and geometric nonlinearity.

## 2 Optimization Process

### 2.1 Defining the trailing edge model and objective function

As shown in Fig.1, both curves represent two ideal shapes of the trailing edge in the different flying states. One side (*A* point) of the structure is supposed to be fixed, and the other side (*B* point) to be sliding horizontally.

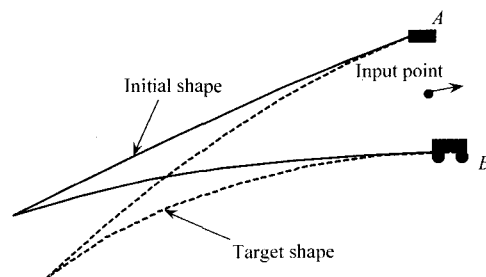


Fig.1 Initial shape and target shape.

Firstly, the design domain should be defined by the initial curve shape, the input location and the boundary conditions. Then, it is divided with a beam element network simulating the bird's feather as shown in Fig.2. This is termed the partial ground structure method.

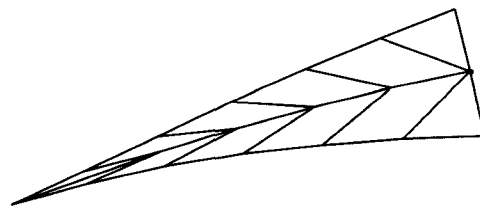


Fig.2 Discretization of the design domain.

The simplest and most effective way to manufacture the planar compliant mechanism is to use wire-cutting technology. In the optimization program, all the elements are of rectangular beams with

the same width equal to the thickness of the material, every beam's height being a design variable.

In order to make the structure's deformation come close to the target shape curve, the least square error (LSE) between the deformed curve and the target curve is defined as the objective function. LSE is the sum of squares of position differences of various points along the curves<sup>[5]</sup>. Its expression is

$$\min f(x) = \sum_{i=1}^p \sqrt{(x_{\text{Tar},i} - x_{\text{Def},i})^2 + (y_{\text{Tar},i} - y_{\text{Def},i})^2} \quad (1)$$

where  $i$  ( $=1, 2, \dots, p$ ) is the number of the points along the curves,  $p$  is the total number of points.  $(x_{\text{Tar},i}, y_{\text{Tar},i})$  and  $(x_{\text{Def},i}, y_{\text{Def},i})$  are the coordinates of  $i$ th node on the target and deformed boundary curve respectively.

The constraints are

$$h_{\min} < h_i < h_{\max} \quad (2)$$

$$h_{\min} < h_b < h_{\max} \quad (3)$$

$$d_{\max} \leq [d] \quad (4)$$

$$\sigma_{\max} \leq \sigma_s \quad (5)$$

$$T_j \in \{0, 1\} \quad (6)$$

where  $j$  ( $=1, 2, \dots, m$ ) is the number of elements,  $m$  is the total number of elements,  $h_i$  the dimension variable,  $h_{\min}$  and  $h_{\max}$  are the lower and upper bounds of the element beam height for all elements with the value dependent on manufacturing,  $h_b$  the height of the boundary elements,  $d_{\max}$  the maximum nodal deformation of the nodes on the curve boundary when the input point is inactive, and should be smaller than  $[d]$  to ensure structure stiffness,  $[d]$  the allowable maximum displacement when the input point is inactive,  $\sigma_{\max}$  the maximum stress of all the elements which must be smaller than  $\sigma_s$  to prevent yielding,  $T_j$  the topology variable equal to 1, or else 0 when the element is eliminated.

## 2.2 GA optimization

GA is an optimization method which simulates the heuristic selection rule in nature, where the fittest living things have the most chance to survive, but the inferior ones also have the opportunity to exist<sup>[14]</sup>. Different from the continuous optimization

method, it does not require the gradient-based information of the objective function.

Every element could be expressed as a topology variable and a dimension variable<sup>[5]</sup>. Therefore, each individual could be coded as follows

$$U = [T_1 \quad T_2 \quad \dots \quad T_i \quad \dots \quad T_{n-1} \quad T_n \quad h_1 \quad h_2 \quad \dots \quad h_i \quad \dots \quad h_{n-1} \quad h_n \quad h_b] \quad (7)$$

where  $n$  is the number of elements except the boundary ones. With the same heights, the boundary elements throughout the optimizing process are represented by only one variable,  $h_b$ .

The fitness is the criterion of the GA optimization. It could be transformed from the objective function into<sup>[12]</sup>

$$\text{Fit}(f(x)) = \exp(-\beta \cdot f(x)) \quad (8)$$

where  $\beta$  is a coefficient deciding the compulsive selection of the better individual. The smaller the value, the more different would be between the two individuals' fitness thus increasing the compulsiveness of choosing the individual of higher fitness.

The selection of control parameters plays an important role in the convergence of the GA. Generally speaking, the cross probability ranges 0.40-0.99; the mutation probability is 0.000 01-0.01, and the number of individuals 10-200.

The variable would be updated through the crossover and mutation, so the possible design could generate in the GA process.

## 2.3 Finite element analysis (FEA)

Because of the limited design variables and the target function, the optimization module of FEA software could not be used to design the compliant morphing mechanism. Therefore, this paper programmed the GA in MATLAB and the FEA in ANSYS. In the FEA, taking only account of geometric nonlinearities and the material being of linear elasticity, ANSYS could solve the node displacements and the element stresses. Then by deleting the elements with low stress, the fitness could be calculated. Fig.3 shows the detailed process.

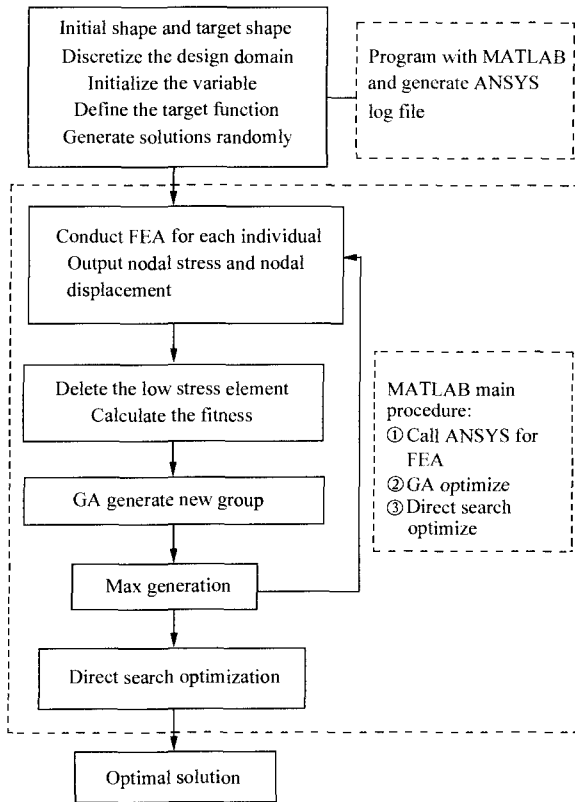


Fig.3 Flowchart of the structural optimization program.

### 2.4 Second optimization

Although the GA could optimize the topology and dimension simultaneously in a large solution space, the dimension usually could not directly converge to the optimization. In order to solve this problem, after the GA, the Direct Search method should be used to find the best values of the input displacement and the dimensions of the elements which remain in the results after the GA.

For morphing of compliant mechanism, Fig.3 describes the whole optimization process. It mainly contains initialization of the design domain, FEA, GA optimization and second optimization.

### 3 Presentation of Results

Adopted from Ref.[5], the sizes of the initial and the target trailing edge are reduced by sixty percent. Table 1 lists the design parameters.

Because the displacement is used as the input, the nonlinear analysis could hardly converge and the stress of the initial solutions is very large, which should be considered after thirtieth generation.

Table 1 Design parameters

Parameters	Values
Material	Aluminous alloy (2A12-T4)
Thickness/mm	25
Plastic modulus/GPa	71
Poisson ratio	0.33
Yield stress/MPa	379
[d]/mm	10
$h_{min}$ /mm	1
$h_{max}$ /mm	5
Input displacement/mm	10 (11°)
Distributed pressure loads/(N·mm <sup>-1</sup> )	1
Max generation	50
Cross probability	0.8
Mutate probability	0.02

Fig.4 and Fig.5 illustrate the results from the GA optimization and the second optimization respectively.

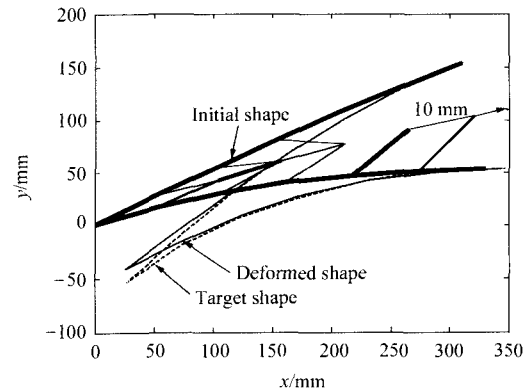


Fig.4 Results after the GA optimization.

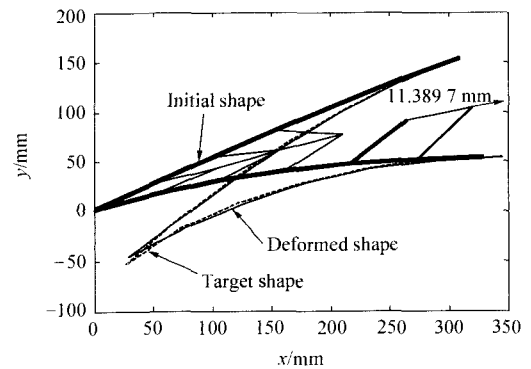


Fig.5 Results after the second optimization.

From Table 2, it could be found that through the second optimization of the input displacement and the dimension, the LSE is reduced by 1.352 8

mm and improved by 3.13%. The altered angle is increased by 1.049 3°.

**Table 2 Results after the two optimizations**

Results	GA result	Second result
Input displacement/mm	10.000 0	11.389 7
LSE/mm	43.174 1	41.821 3
Max stress/MPa	262.548 8	274.669 1
Max displacement/mm	0.309 6	0.341 3
Deformed angle/(°)	8.274 9	9.324 3

Fig.6 shows the influences of the parameters when the outside distributed pressure load changes from 0 to 10 N/mm and the input displacement remains 11.389 7 mm on the optimal structure. It could be seen that the optimal structure has a good stability if the load is kept in the range of 0-5 N/mm. As the external load exceeds 5 N/mm, the max stress is likely to exceed the yield stress.

Because this optimization program is based on the MATLAB and ANSYS, in order to verify the results, an attempt is made to introduce the analyti-

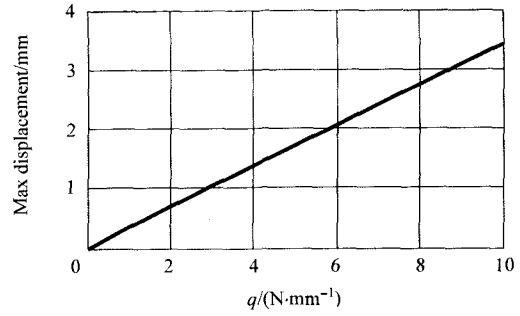
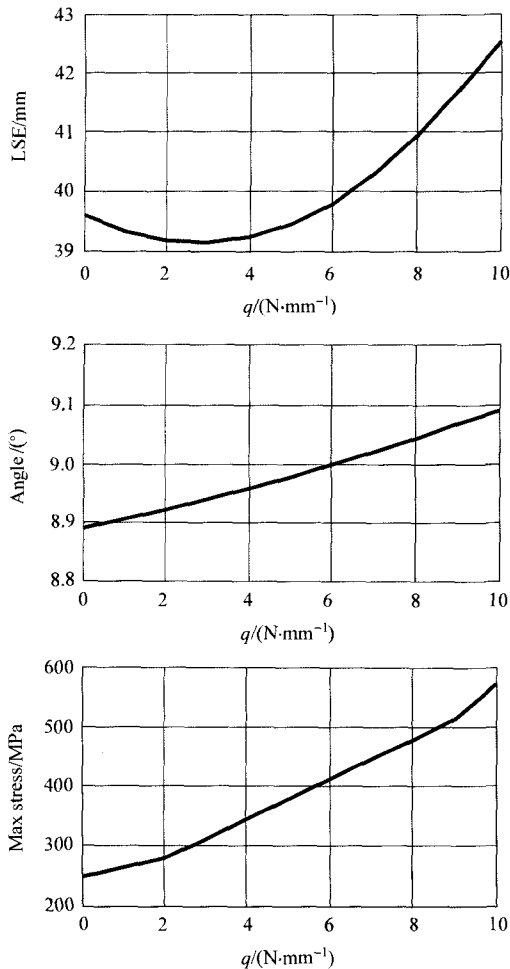


Fig.6 Stability of final optimal structure.

cal results of the optimized structure into ANSYS and PATRAN respectively, and then a comparison is made between them. As shown in Fig.7 and Fig.8, the two altered shapes are in good agreement: for in ANSYS the tip displacement is 54.97 mm and in PATRAN 54.50 mm. The minor difference between them is from the software.

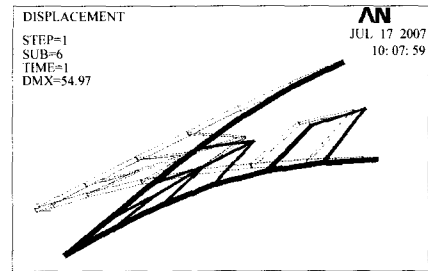


Fig.7 Results of FEA in ANSYS.

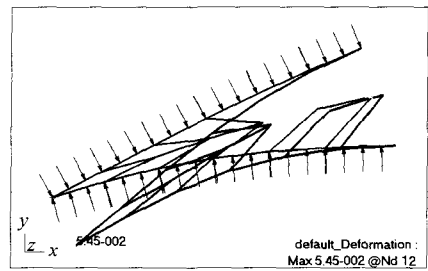


Fig.8 Results of FEA in PATRAN.

On the other hand, a model is made by wire-cutting technology to verify the analytical results. The material of the model, identical with that of the design, is 5 mm thick. In the experiment, the distributed pressure load is assumed to be zero, the input displacement 11.389 7 mm with the required input load 146 N. Fig.9 shows the model and the experimental result. The altered angle is measured 9.3°, and the tip displacement 53 mm. The altered shape well accords to the optimized result. If a dis-

placement of 11.389 7 mm is imposed on the model, the theoretical tip displacement is 54.796 mm. Because of the friction there is between the model and the experiment table, a tiny difference will take place between the measured data and the calculated results.

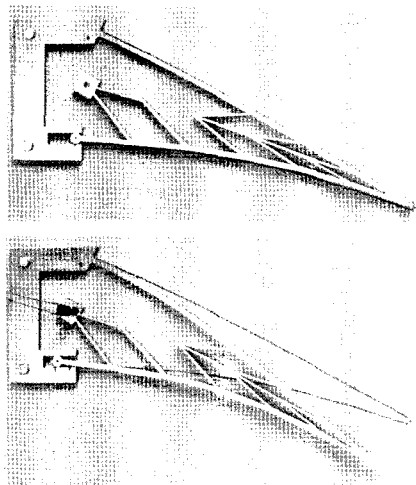


Fig.9 The model and experimental result.

#### 4 Conclusions

Proved by the simulation and experiments, the proposed method to design morphing compliant mechanism is effectual in turning out a trailing edge with required morphing effects and ability of withstanding external loads. The combination of MATLAB and ANSYS in the optimization renders the program simple and universal. There is no need for frequent changes of the rigid matrix. It also avoids the complexity of programming the nonlinear FEA and the transforming distributed loads into nodal loads. Using the mixed code, the topology and the dimension could simultaneously be optimized by the GA. Removing the free elements after the FEA could speed up the optimization. The second optimization could improve the GA results.

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