Int J Adv Manuf Technol (2004) 24: 789–793
DOI 10.1007/s00170-003-1741-8

ORIGINAL ARTICLE

Feng Xianying · Wang Aiqun · Linda Lee

Study on the design principle of the LogiX gear tooth profile and the selection of its inherent basic parameters

Received: 2 January 2003 / Accepted: 3 March 2003 / Published online: 3 November 2004

Abstract The development of scientific technology and productivity has called for increasingly higher requirements of gear transmission performance. The key factor influencing dynamic gear performance is the form of the meshed gear tooth profile. To improve a gear's transmission performance, a new type of gear called the *LogiX* gear was developed in the early 1990s. However, for this special kind of gear there remain many unknown theoretical and practical problems to be solved. In this paper, the design principle of this new type of gear is further studied and the mathematical module of its tooth profile deduced. The influence on the form of this type of tooth profile and its mesh performance by its inherent basic parameters is discussed, and reasonable selections for LogiX gear parameters are provided. Thus the theoretical system information about the LogiX gear are developed and enriched. This study impacts most significantly the improvement of load capacity, miniaturisation and durability of modern kinetic transmission products.

Keywords Basic parameter · Design principle · LogiX gear · Minute involute · Tooth profile

1 Introduction

In order to improve gear transmission performance and satisfy some special requirements, a new type of gear [1] was put forward; it was named "LogiX" in order to improve some demerits of W-N (Wildhaver-Novikov) and involute gears.

Besides having the advantages of both kinds of gears mentioned above, the new type of gear has some other excellent characteristics. On this new tooth profile, the continuous concave/convex contact is carried out from its dedendum to its addendum, where the engagements with a relative curvature of zero are assured at many points. Here, this kind of point is called the null-point (N-P). The presence of many N-Ps during the mesh process of LogiX gears can result in a smaller sliding coefficient, and the mesh transmission performance becomes almost rolling friction accordingly. Thus this new type of gear has many advantages such as higher contact intensity, longer life and a larger transmission-ratio power transfer than the standard involute gear. Experimental results showed that, given a certain number of N-Ps between two meshed LogiX gears, the contact fatigue strength is 3 times and the bend fatigue strength 2.5 times larger than those of the standard involute gear. Moreover, the minimum tooth number can also be decreased to 3, much smaller than that of the standard involute gear.

The LogiX gear, regarded as a new type of gear, still presents some unsolved problems. The development of computer numerical controlling (CNC) technology must also be taken into consideration new high-efficiency methods to cut this new type of gear. Therefore, further study of this new type of gear most significantly impacts the acceleration of its broad and practical application. This paper has the potential to usher in a new era in the history of gear mesh theory and application.

2 Design principle of LogiX tooth profile

According to gear mesh and manufacturing theories, in order to simplify problem analysis, generally a gear's basic rack is begun with some studies [2]. So here let us discuss the basic rack of the LogiX gear first. Figure 1 shows the design principle of divided involute curves of the LogiX rack. In Fig. 1, $P.L$ represents a pitch line of the LogiX rack. One point O_1 is selected to form the angle $\bullet n_0 O_1 N_1 = \alpha_0$, $P.L \bullet O_1 N_1$. The points of intersection by two radials $O_1 n_0$ and $O_1 N_1$ and the pitch line $P.L$ are N_1 and n_0. Let $O_1 n_0 = G_1$, extend $O_1 n_0$ to $O_1^{'}$, and make two tangent basic circles whose centres are O_1, $O_1^{'}$ and radii are equal to G_1.. The point of intersection between circle O_1 and pitch line

F. Xianying (✉) · W. Aiqun
School of Mechanical Engineering,
Shandong University,
P.R. China
E-mail: FXYing@sdu.edu.cn
Tel.: +86-531-8395852(0)

L. Lee
School of Mechanical & Manufacturing Engineering,
Singapore Polytechnic,
Singapore

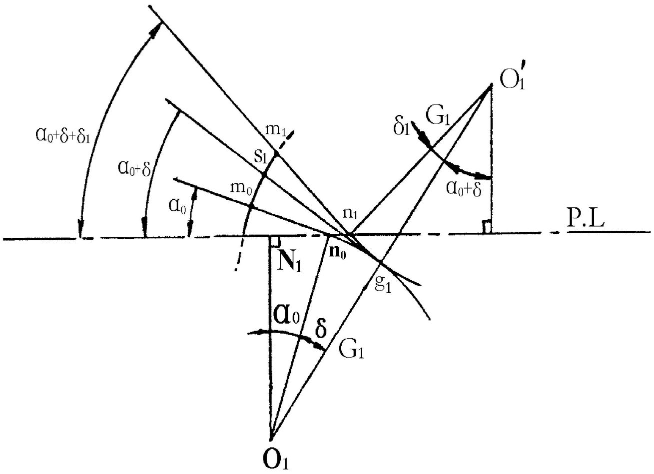


Fig. 1. Design principle of LogiX rack tooth profile

$P.L$ is n_0. The point of intersection between circle O_2 and pitch line $P.L$ is n_1. Make the common tangent g_1s_1 of basic circle O_1 and O_1', then generate two minute involute curves m_0s_1 and s_1m_1 whose basic circle centres are O_1 and O_1'. The radii of curvature at points m_0 and m_1 on the tooth profile should be: $\rho_{m0} = m_0n_0$, $\rho_{m1} = m_1n_1$, and the centres are met on the pitch line.

Multiple different minute involutes consisting of a LogiX profile should be arranged for a proper sequence. The pressure angle of the next minute involute curve m_1m_2 should have an increment comparable to its last segment m_0m_1. The centres of curvature at extreme points m_1, m_2, etc. should be on the pitch line, and the radius of the basic circle is a function of pressure [1] – it varies from G_1 to G_2. The condition for joining front and rear curves is that the radius of curvature at point m_1 must be equal to the radius of curvature just after point m_1, and the radius of curvature at point m_2 must be equal to the radius of curvature just after point m_2. Figure 2 shows the connection and process of generating minute involute curves. According to the above discussion, the whole tooth profile can be formed.

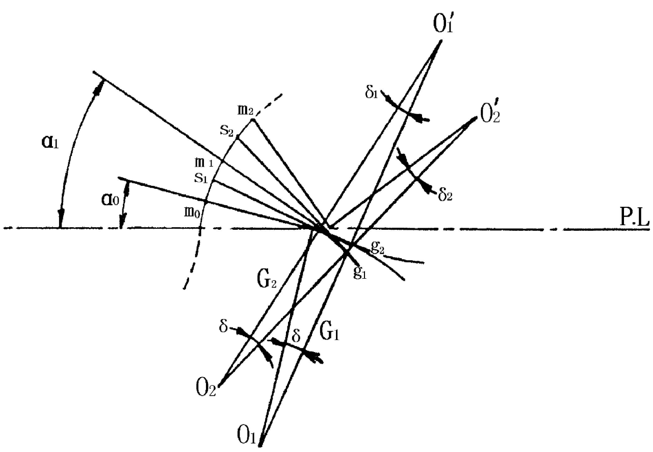


Fig. 2. Connection of minute involute curves

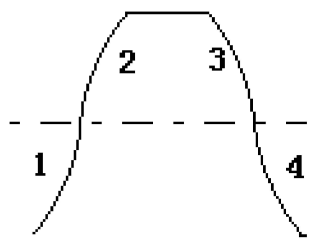
3 Mathematic module of LogiX tooth profile

3.1 Mathematic module of the basic LogiX rack

According to the above-mentioned design principle, the curvature centre of every finely divided profile curve should be located at the rack pitch line, and the value of the relative curvature at every point connecting different minute involute curves should be zero. The design of the tooth profile is symmetrical with respect to the pitch line, and the addendum is convex while the dedendum is concave. Thus for the whole LogiX tooth profile, it can be dealt with by dividing it into four parts, as shown in Fig. 3. Set up the coordinates as shown in Fig. 4, where the origin of the coordinates O coincides with the point of intersection m_0 between rack pitch line $P.L$ and the initial divided minute involute curve.

According to the coordinates set up in Fig. 4, the formation of initial minute involute curve m_0m_1 is shown in Fig. 5.

Fig. 3. LogiX rack tooth profile



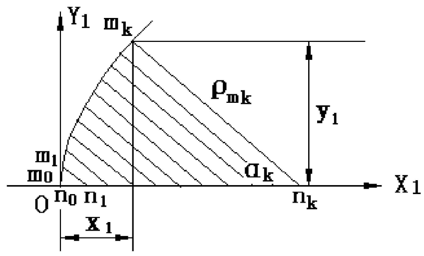


Fig. 4. Set-up of coordinates

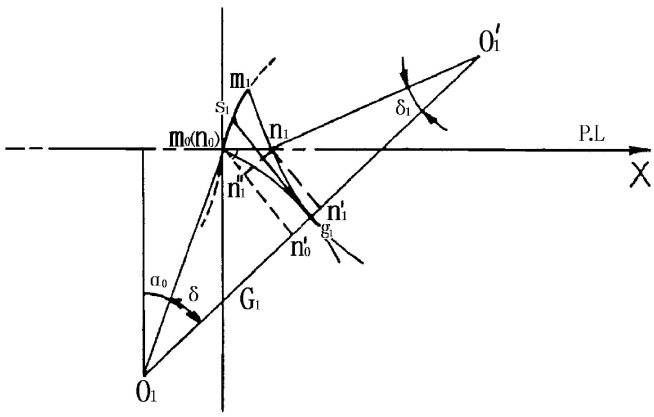


Fig. 5. Formation process of initial minute involute curve m_0m_1

Here: $n_0n_0' \bullet O_1O_1'$, $n_1n_1' \bullet O_1O_1'$, $n_1n_1 \bullet n_0n_0'$, and the parameters α_0, δ, G_1 and ρ_{m0} are given as initial conditions. The curvature radius of the involute curve at point s_1 is $\rho_{s1} = G_1\delta$, or $\rho_{s1} = \rho_{m1} + G_1\delta_1$. Thus the curvature radius and pressure angle of the minute involute curve at point m_1 are as follows:

$$\rho_{m1} = \rho_{s1} - G_1\delta_1 = G_1(\delta - \delta_1) \tag{1}$$

$$\alpha_1 = \alpha_0 + \delta + \delta_1 \,. \tag{2}$$

According to the geometrical relationship, we can deduce:

$$tg(\alpha_0 + \delta) = \frac{2G_1 - G_1\cos\delta - G_1\cos\delta_1}{G_1\sin\delta - G_1\sin\delta_1} = \frac{2 - (\cos\delta + \cos\delta_1)}{\sin\delta - \sin\delta_1} \,. \tag{3}$$

Based on Eqs. 1, 2 and 3 and the forming process of the LogiX rack profile, the curvature radius formula of an arbitrary point on the profile is deduced: $\rho_{mi} = \rho_{mi-1} + G_i(\delta - \delta_i)$. When $i = k$ and $\rho_{m0} = 0\bullet$, it is expressed as follows:

$$\rho_{mk} = G_1(\delta - \delta_1) + G_2(\delta - \delta_2) + \cdots + G_k(\delta - \delta_k) = \sum_{i=1}^{k} G_i(\delta - \delta_i) \,. \tag{4}$$

Similarly, the pressure angle on an arbitrary k point of the tooth profile can be deduced as follows:

$$\alpha_k = \alpha_0 + (\delta + \delta_1) + (\delta + \delta_2) + \cdots (\delta + \delta_k) = \alpha_0 + \sum_{i=1}^{k} (\delta + \delta_i) = \alpha_0 + k\delta + \sum_{i=1}^{k} \delta_i \,. \tag{5}$$

By $n_{i-1}n_i = G_i(\sin\delta - \sin\delta_i)/\cos(\alpha_{i-1} + \delta)$, Eq. 5 can be obtained:

$$n_0n_k = \sum_{i=1}^{k} n_{i-1}n_i = \sum_{i=1}^{k} \frac{G_i(\sin\delta - \sin\delta_i)}{\cos(\alpha_{i-1} + \delta)}. \tag{6}$$

Thus the mathematical model of the No. 2 portion for the LogiX rack profile is as follows:

$$\begin{cases} x_1 = n_0n_k - \rho_{mk}\cos\alpha_k \\ y_1 = \rho_{mk}\sin\alpha_k \end{cases} \text{(No. 2)} \,. \tag{7}$$

Similarly, the mathematical models of the other three segments can also be obtained as follows:

$$\begin{cases} x_1 = -(n_0n_k - \rho_{mk}\cos\alpha_k) \\ y_1 = -\rho_{mk}\sin\alpha_k \end{cases} \text{(No.1)} \tag{8}$$

$$\begin{cases} x_1 = s - (n_0n_k - \rho_{mk}\cos\alpha_k) \\ y_1 = \rho_{mk}\sin\alpha_k \end{cases} \text{(No.3)} \tag{9}$$

$$\begin{cases} x_1 = s + n_0n_k - \rho_{mk}\cos\alpha_k \\ y_1 = -\rho_{mk}\sin\alpha_k \end{cases} \text{(No.4)} \,. \tag{10}$$

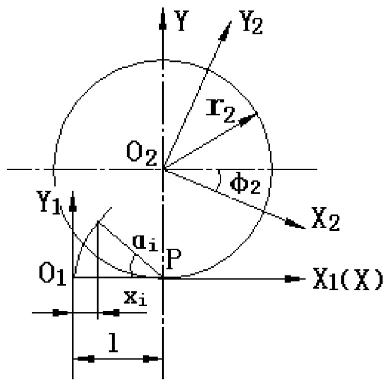


Fig. 6. Mesh coordinates of LogiX gear and its basic rack

3.2 Mathematical module of the LogiX gear

The coordinates $O_1X_1Y_1$, $O_2X_2Y_2$ and PXY are set up as shown in Fig. 6 to express the mesh relationship between the LogiX rack and the LogiX gear. Here, $O_1X_1Y_1$ is fixed on the rack, and O_1 is the point of intersection between the rack tooth profile and its pitch line. $O_2X_2Y_2$ is fixed on the meshed gear, and O_2 is the gear's centre. PXY is an absolute coordinate, and P is the point of intersection of the rack's pitch line and the gear's pitch circle.

In accordance with gear meshing theories [3], if the above model of the LogiX rack tooth profile is changed from coordinate $O_1X_1Y_1$ to OXY, and then again to $O_2X_2Y_2$, a new type of gear profile model can be deduced as follows:

$$\begin{cases} x_2 = -\rho_{mk}\cos\alpha_k\cos\varphi_2 - (\rho_{mk}\sin\alpha_k - r_2)\sin\varphi_2 \\ y_2 = -\rho_{mk}\cos\alpha_k\sin\varphi_2 + (\rho_{mk}\sin\alpha_k - r_2)\cos\varphi_2 \,. \end{cases} \tag{11}$$

Here the positive direction of φ_2 is clockwise, and only the model of the LogiX gear tooth profile in the first quadrant of the coordinates is given.

4 Effect on the performance of the LogiX gear by its inherent parameters and their reasonable selection

Besides the basic parameters of the standard involute rack, the LogiX tooth profile has inherent basic parameters such as initial pressure angle α_0, relative pressure angle δ, initial basic circle radius G_0, etc. The selection of these parameters has a great influence on the form of the LogiX tooth profile, and the form directly influences gear transmission performance. Thus the reasonable selection of these basic parameters is very important.

4.1 Influence and selection of initial pressure angle α_0

Considering the higher transmission efficiency in practical design, the initial pressure angle α_0 should be selected as 0°. But the final calculation result showed that the LogiX gear tooth profile cut by the rack tool whose initial pressure angle was equal to zero would be overcut on the pitch circle generally. Thus the initial pressure angle α_0 cannot be zero. Comparing the relative double circle-arc gear [3], we can also deduce that the smaller the initial pressure angle α_0, the larger the gear number for producing the overcut. Thus the initial pressure angle α_0 should not only not be zero, but should not be too small, either. From Eqs. 3, 4 and 5, the influence of α_0 on the LogiX tooth profile can be directly described by Fig. 7. Obviously, increasing the initial pressure angle will cause the curvature of the LogiX rack tooth profile to become larger. If the rack selects a larger module and too small an initial pressure angle α_0, its addendum will become too narrow or even overcut. Thus the LogiX tooth profile that selects a larger module should select a smaller α_0, and the profile that selects a smaller module should select a larger α_0. Generally, by practical calculation experience, the selected α_0 should be located within a range of $2° \sim 12°$, and the larger the LogiX gear module, the smaller should be its initial pressure angle α_0.

4.2 Influence and selection of initial basic circle radius G_0

According to the empirical formula $G_i = G_0\{1 - \sin(0.6\alpha_i)\}$ [1], there are two parameters affecting the basic circle radius G_i of the LogiX gear at different positions of tooth profile: one is the G_0 and the other is the initial pressure angle α_i. Figure 8 shows the influence of G_0 on the LogiX tooth profile when certain values of parameter α_0 and δ are selected. Obviously, as G_0 increases, the curvature of the new type of gear tooth profile will become smaller and smaller. Conversely, it will become increasingly larger as G_0 decreases. Thus the new type of rack with a large module parameter should select a large G_0 value, and one having a small module parameter should select a small G_0 value.

4.3 Influence and selection of relative pressure angle δ

Figure 9 shows the variable of the tooth profile affected by the δ parameter. According to the forming process of the LogiX tooth profile, the smaller the selected parameter δ, the larger the number of N-Ps meshing on the tooth profile of two LogiX gears. From Sect. 2.1 the formula describing the relative pressure angle δ_k of an arbitrary N-P m_k can be deduced as follows:

$$\frac{\sin(\alpha_{k-1}+\delta)}{\cos(\alpha_{k-1}+\delta)} = \frac{2-(\cos\delta+\cos\delta_k)}{\sin\delta-\sin\delta_k}\,. \tag{12}$$

By Eqs. 5 and 12, the larger the δ parameter being selected, the larger will be the δ_k parameter, and at certain selected values of the initial pressure angle and maximum pressure angle, the lower will be the number of N-Ps. By contrast, the smaller the δ parameter, the larger the number of N-Ps. While δ is 0.0006°, the number of zero points can exceed 46,000. In this case, selecting a gear module of $m = 100$, the length of the micro-involute curve between two adjoining N-Ps will be only a few microns. That is to say, during the whole meshing process of the LogiX gear transmission, the sliding and rolling motions happen alternately and last only a few micro-seconds from one motion to another between two meshed gear tooth profiles. The greater the number of N-Ps, the longer the relative rolling time between two LogiX gears and the shorter the relative sliding time between two LogiX gears. Thus abrasion of the gear decreases and its loading capability and life span are improved. But, considering the restriction of memory capability, interpolation speed, angular resolution, etc. for the CNC machine tool used while cutting this type of gear, the relative pressure angle selected should not be very small. $\delta \geqslant 0.0006°$ is generally satisfactory.

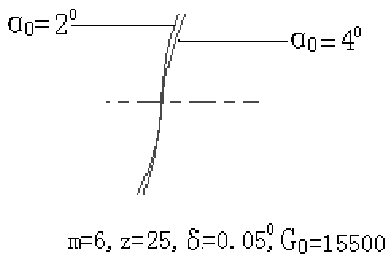


Fig. 7. Influence of α_0 on LogiX tooth profile

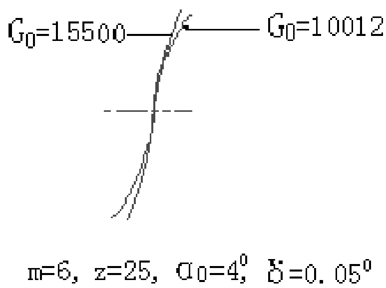


Fig. 8. Influence of G_0 on LogiX tooth profile

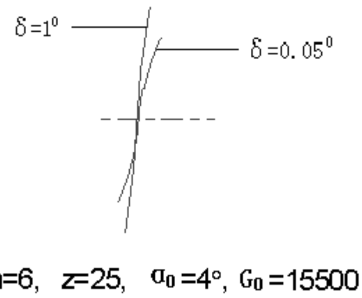


Fig. 9. Influence of δ on LogiX tooth profile

Table 1. Parameter values selected for LogiX rack at different modules

m(mm)	α_0	δ	G_0(mm)
1	10°	0.05°	6000
2	8.0°	0.05°	9500
4	6.0°	0.05°	10000
5	5.0°	0.05°	11000
6	4.0°	0.05°	12000
8	3.2°	0.05°	12024
10	2.8°	0.05°	14000
12	2.6°	0.05°	16500
15	2.5°	0.05°	20024
18	2.4°	0.05°	30036
20	2.4°	0.05°	35000
22	2.3°	0.05°	38000

4.4 Reasonable selection example

Based on the above analytical rules for LogiX gear inherent parameter selection, a reasonable calculation and selection results for the initial pressure angle and basic circle radius while selecting different modules at the relative pressure angle $\delta = 0.05°$ are listed in Table 1 for reference. In fact, the practical selections should be reasonably modified by the concrete cutting conditions and the special purpose requirement.

5 Conclusions

The following conclusions were made based on the findings presented in this paper.

1. Two-dimensional meshing transmission models of LogiX gears were deduced by further analysis of its forming principle.
2. The influence on the LogiX gear tooth profile and its performance by the gear's own basic parameters such as initial pressure angle, initial basic circle radius and relative pressure angle was discussed and their reasonable selection was given.
3. The theoretical system of the LogiX gear was developed and the mathematical basis for generating the LogiX tooth profile by modern CNC technology was established. The characteristics of the LogiX gear, which are different from those of the ordinary standard involute gear, can have broad application and most significantly impact the improvement of carrying capacity, miniaturisation and longevity of kinetic transmission products.

References

1. Komori T, Arga Y, Nagata S (1990) A new gear profile having zero relative curvature at many contact points. Trans ASME 112(3): 430–436
2. Xutang W (1982) Gear meshing theory. Machinery Industry Press, Beijing
3. Jiahui S (1994) Circle-arc gears. Machinery Industry Press, Beijing

6 Nomenclature

α_0 initial pressure angle
α_i pressure angle at contact point m_i
δ parameter of pressure angle
ρ_{s1} radius of curvature of gear tooth profile at contact point s_1
ρ_{mi} radius of curvature of gear tooth profile at contact point m_i
ρ_{m1} radius of curvature of gear tooth profile at contact point m_1
G_0 initial radius of basic circle in tooth profile
G_i radius of basic circle of m_i point in gear tooth profile
φ_2 rotation angle of LogiX gear meshing with basic LogiX rack
r_2 radius of basic circle of LogiX gear meshing with basic LogiX rack
m model of gear
z gear tooth number
s gear tooth thickness at pitch circle; here, i is an optional number