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Five-axis milling machine tool kinematic chain design and analysis

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Abstract

Five-axis CNC machining centers have become quite common today. The kinematics of most of the machines are based on a rectangular Cartesian coordinate system. This paper classifies the possible conceptual designs and actual existing implementations based on the theoretically possible combinations of the degrees of freedom. Some useful quantitative parameters, such as the workspace utilization factor, machine tool space efficiency, orientation space index and orientation angle index are defined. The advantages and disadvantages of each concept are analyzed. Criteria for selection and design of a machine configuration are given. New concepts based on the Stewart platform have been introduced recently in industry and are also briefly discussed.

Keywords: Five-axis; Machine tool; Kinematic chain; Workspace; CNC; Rotary axis

1. Introduction

The main design specifications of a machine tool can be deduced from the following principles:

- The kinematics should provide sufficient flexibility in orientation and position of tool and part.
- Orientation and positioning with the highest possible speed.
- Orientation and positioning with the highest possible accuracy.
- Fast change of tool and workpiece.
- Save for the environment.
- Highest possible material removal rate.

The number of axes of a machine tool normally refers to the number of degrees of freedom or the number of independent controllable motions on the machine slides. The ISO axes nomenclature recommends the use of a right-handed coordinate system, with the tool axis corresponding to the *Z*-axis. A three-axis milling machine has three linear slides *X*, *Y* and *Z* which can be positioned everywhere within the travel limit of each slide. The tool axis direction stays fixed during machining. This limits the flexibility of the tool orientation relative to the workpiece and results in a number of different set ups. To increase the flexibility in possible tool workpiece orientations, without need of re-setup, more degrees of freedom must be added. For a conventional three linear axes machine this can be achieved by providing rotational slides. Fig. 1 gives an example of a five-axis milling machine.

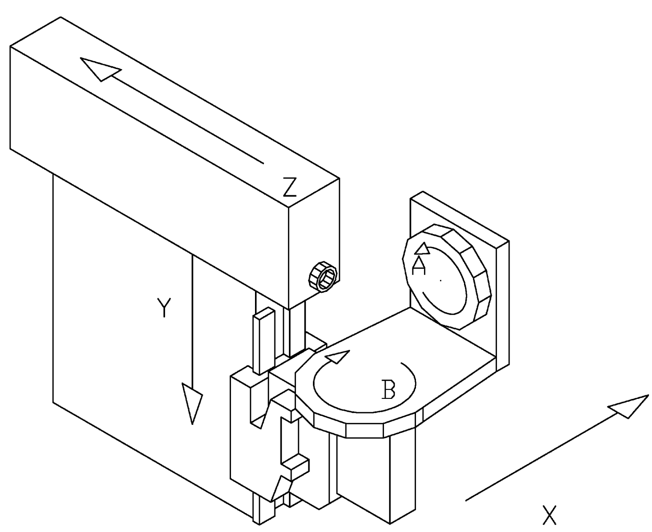

Fig. 1. Five-axis machine tool.

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2. Kinematic chain diagram

To analyze the machine it is very useful to make a kinematic diagram of the machine. From this kinematic (chain) diagram two groups of axes can immediately be distinguished: the workpiece carrying axes and the tool carrying axes. Fig. 2 gives the kinematic diagram of the five-axis machine in Fig. 1. As can be seen the workpiece is carried by four axes and the tool only by one axis.

The five-axis machine is similar to two cooperating robots, one robot carrying the workpiece and one robot carrying the tool.

Five degrees of freedom are the minimum required to obtain maximum flexibility in tool workpiece orientation, this means that the tool and workpiece can be oriented relative to each other under any angle. The minimum required number of axes can also be understood from a rigid body kinematics point of view. To orient two rigid bodies in space relative to each other 6 degrees of freedom are needed for each body (tool and workpiece) or 12 degrees. However any common translation and rotation which does not change the relative orientation is permitted reducing the number of degrees by 6. The distance between the bodies is prescribed by the toolpath and allows elimination of an additional degree of freedom, resulting in a minimum requirement of 5 degrees.

3. Literature review

One of the earliest (1970) and still very useful introductions to five-axis milling was given by Baughman [1] clearly stating the applications. The APT language was then the only tool to program five-axis contouring applications. The problems in postprocessing were also clearly stated by Sim [2] in those earlier days of numerical control and most issues are still valid. Boyd in Ref. [3] was also one of the early introductions. Beziers' book [4] is also still a very useful introduction. Held [5] gives a very brief but enlightening definition of multi-axis machining in his book on pocket milling. A recent paper applicable to the problem of five-axis machine workspace computation is the multiple sweeping using the Denawit-Hartenberg representation method developed by Abdel-Malek and Othman [6].

Many types and design concepts of machine tools which can be applied to five-axis machines are discussed in Ref. [7] but not specifically for the five-axis machine.

The number of setups and the optimal orientation of the part on the machine table is discussed in Ref. [8]. A review about the state of the art and new requirements for tool path generation is given by B.K. Choi et al. [9]. Graphic simulation of the interaction of the tool and workpiece is also a very active area of research and a good introduction can be found in Ref. [10].

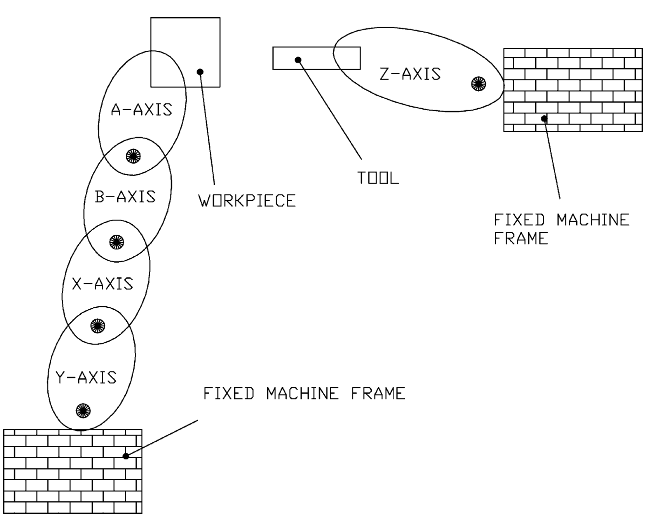

Fig. 2. Kinematic chain diagram.

4. Classification of five-axis machines' kinematic structure

Starting from Rotary (R) and Translatory (T) axes four main groups can be distinguished: (i) three T axes and two R axes; (ii) two T axes and three R axes; (iii) one T axis and four R axes and (iv) five R axes. Nearly all existing five-axis machine tools are in group (i). Also a number of welding robots, filament winding machines and laser machining centers fall in this group. Only limited instances of five-axis machine tools in group (ii) exist for the machining of ship propellers. Groups (iii) and (iv) are used in the design of robots usually with more degrees of freedom added.

The five axes can be distributed between the workpiece or tool in several combinations. A first classification can be made based on the number of workpiece and tool carrying axes and the sequence of each axis in the kinematic chain. Another classification can be based on where the rotary axes are located, on the workpiece side or tool side. The five degrees of freedom in a Cartesian coordinates based machine are: three translatory movements X,Y,Z (in general represented as TTT) and two rotational movements AB, AC or BC (in general represented as RR).Combinations of three rotary axes (RRR) and two linear axes (TT) are rare. If an axis is bearing the workpiece it is the habit of noting it with an additional accent. The five-axis machine in Fig. 1 can be characterized by $X'Y'A'B'Z$. The $XYAB$ axes carry the workpiece and the Z-axis carries the tool. Fig. 3 shows a machine of the type $XYZA'B'$, the three linear axes carry the tool and the two rotary axes carry the workpiece.

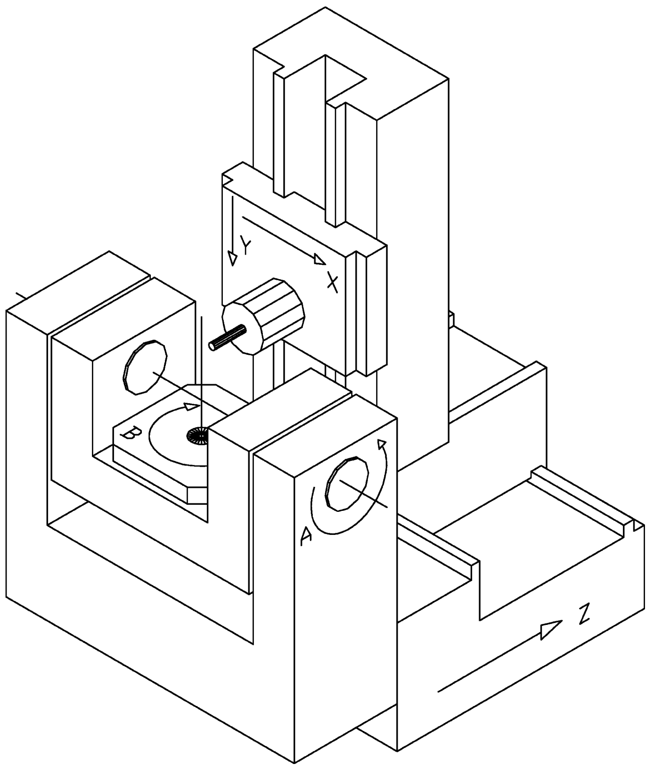

Fig. 3. *XYZA′B′* machinery.

4.1. Classification based on the sequence of workpiece and tool carrying axes

Theoretically the number of possible configurations is quite large if the order of the axes in the two kinematic chains of the tool and workpiece carrying axes is counted as a different configuration. Also the combinations with only two linear axes and three rotary axes are included.

One tool carrying axis and four workpiece carrying axes can be combined in a five-axis machine as follows: for each possible tool carrying axis *X*,*Y*,*Z*,*A*,*B*,*C* the other four workpiece carrying axes can be selected from the five remaining axes. So the number of combinations of four axes out of five with considering different permutation as another configuration is 5×4!=120 for each possible tool axis selection (1 out of 6 or 6 possibilities). So theoretically there are 6×120=720 possible five-axis machines with one tool carrying axis. The same analysis can be done for all other combinations. With *t* the number of tool carrying axes and *w* the number of workpiece carrying axes ($w+t=5$) the total number of combinations is as follows.

$$N_{comb}=\binom{6}{t}t!\binom{6-t}{w}w! \quad t\leq 3,\ t+w=5 \qquad (1)$$

$$N_{comb}=\binom{6}{w}w!\binom{6-w}{t}t! \quad t>3,\ t+w=5 \qquad (2)$$

The value of this equation is always equal to 6! or 720 when $w+t=5$. Some of these 720 combinations will be containing only two linear axis. If only five-axis machines with three linear axes are considered, only 3×5!=360 combinations are still possible.

The set *Gt* of combinations is characterized by a fixed value of *t*. This set is identical to the set $G'w$ characterized by a fixed value of *w*, $w=5-t$. Using above definitions following subgroups of five-axis machines exist: (i) Group $G0/G'5$; (ii) Group $G1/G'4$; (iii) Group $G2/G'3$; (iv) Group $G3/G'2$; (v) Group $G4/G'1$; (vi) Group $G5/G'0$.

4.1.1. G5/G0′ machine

All axes carry the tool and the workpiece is fixed on a fixed table. Fig. 4 shows a machine with all the five axes carrying the tool. The kinematic chain is *XBYAZ* (*TRTRT*). This machine was one of the earliest models of five-axis machines to handle very heavy workpieces. As there are many links in the tool carrying kinematic chain, there can be a considerable error due to elastic deformations and backlash in the slides.

4.1.2. G0/G5′ machine

All axes carry the workpiece and the tool is fixed in space. This construction is best used for very small workpieces (see Section 6.3).

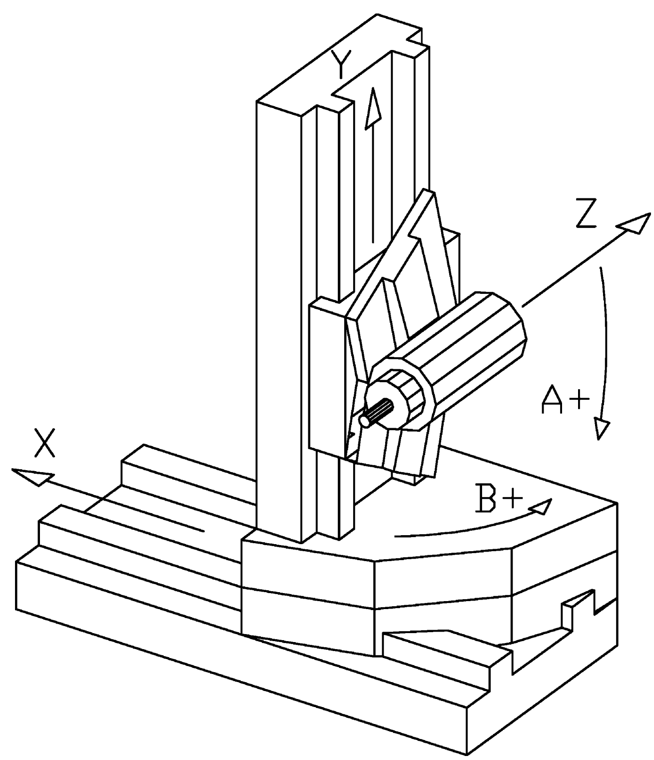

Fig. 4. *XBYAZ* machine.

4.1.3. G4/G1′ machine

Four axes carry the tool and one axis carries the workpiece. There are basically two possibilities, the workpiece carrying axis can be R' or T'.

4.1.4. G1/G4′ machine

One axis carries the tool and the other four axes carry the workpiece. There are basically two possibilities, the single axis kinematic chain can be R or T. Fig. 1 is an example of such a machine, with the single tool carrying axis T.

4.1.5. G3/G2′ machine

Three axes carry the tool and two axes carry the workpiece. There are basically three possibilities, the workpiece carrying axes can be both linear ($T'T'$) both rotational ($R'R'$) or mixed ($T'R'$). Fig. 5 gives an example of a machine with the tool carried by two rotary axes and one linear axis. This machine allows processing of large workpieces but the construction of the toolside is complicated. The most common configuration is the workpiece carried by the two rotary axes such as the one given in Figs. 3, 6 and 8.

4.1.6. G2/G3′ machine

Two axes carry the tool and three axes carry the workpiece. There are basically three possibilities, the tool carrying axis can be both linear (TT) both rotational (RR) or mixed (TR). Fig. 7 shows the mixed construction. Fig. 8 shows two linear axes carrying the tool.

4.2. Classification based on the location of rotary axes

The machines can be classified depending on the place where the rotation axes are implemented.

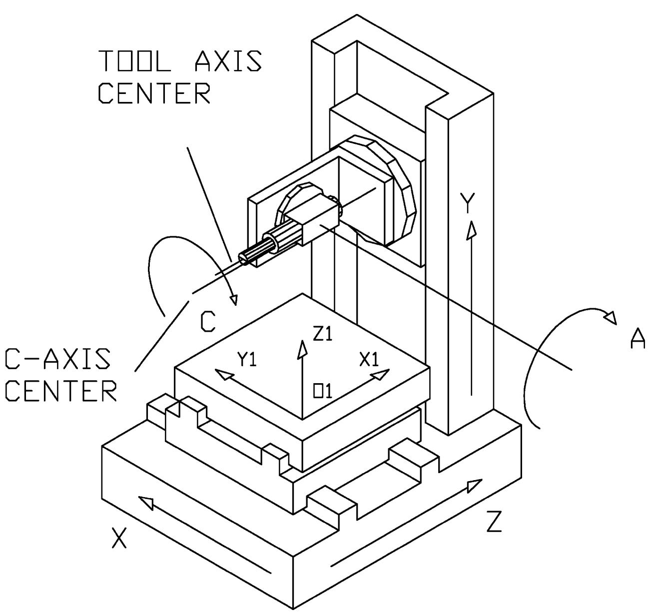

Fig. 5. *X′Z′CAY* machine.

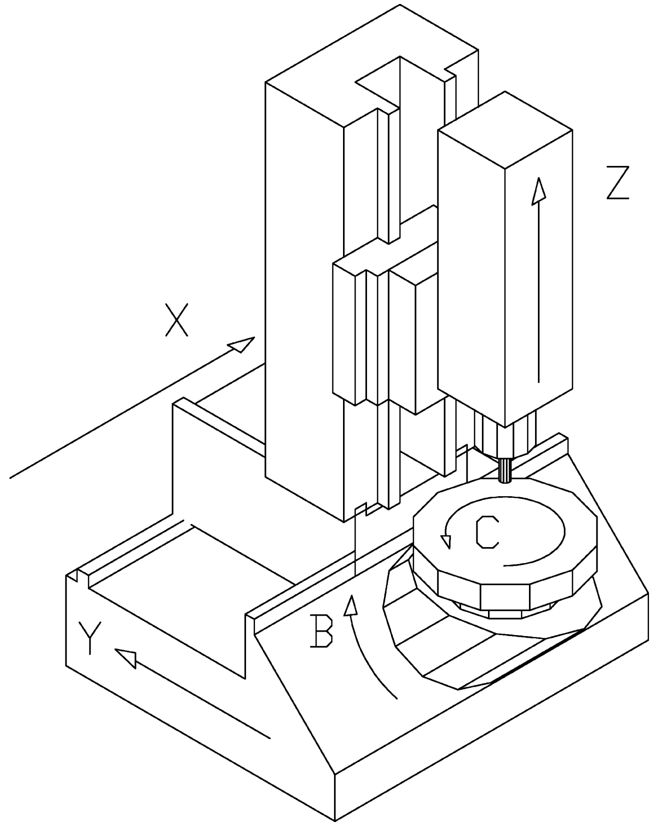

Fig. 6. *B′C′ZYX* machine.

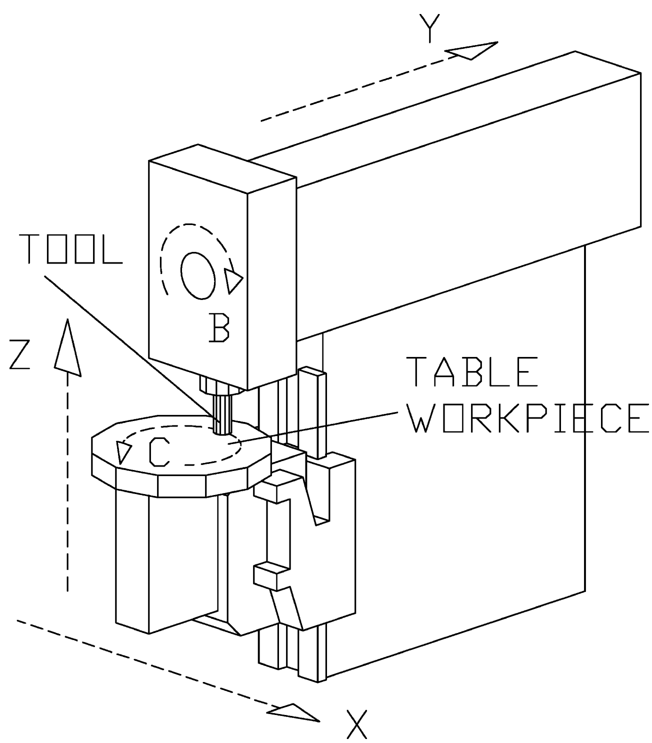

Fig. 7. *Z′X′C′BY* machine.

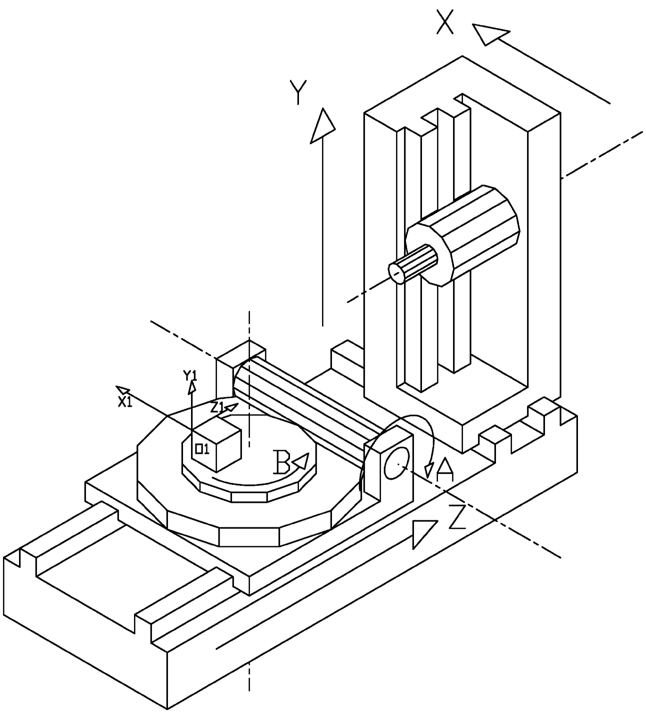

Fig. 8. $Z'A'B'YX$ machine.

Only machines with two rotary axes and three linear axes will be considered further. The possible configurations are:

(a) rotation axes are implemented on tool spindle;
(b) rotation axes are implemented on machine table;
(c) combination of both.

The sequence of the axes in the tool or workpiece carrying kinematic chain is not important if the axes are of the same type R or T. In general, if there are N'_T translatory axes and N'_R rotary axes in the workpiece carrying kinematic chain and N_T translatory axes and N_R rotary axes in the tool kinematic chain, then the numbers of combinations is [11]:

$$N_{comb}=\frac{(N'_T+N'_R)!}{N'_T!N'_R!}\cdot\frac{(N_T+N_R)!}{N_T!N_R} \quad (3)$$

with $N'_T+N_T=3$, $N'_R+N_R=2$

The number of combinations of each group will be given below case by case. The total number of combinations over all groups is 60. From the design point of view this is a more tractable number of alternatives to be considered.

4.2.1. $R'R'$ machine

The two rotary axes carry the workpiece. The tool axis can be fixed or carried by one (T), two (TT) or three (TTT) linear axes.

The number of possible designs is the sum of the following combinations:

(i) For the group $G0/G5'$ the tool is fixed in space all the five axes will carry the workpiece. The number of different designs is 10 ($N_T'=3$ and $N_R'=2$), (Figs. 15 and 16).
(ii) For the group $G1/G4'$, $N_T+N_R=1$, so $N_T=1$ and $N_R=0$, is the only possible choice for the tool kinematic chain. Equation (3) gives $N_{COMB}=6$. The combinations are: $R'R'T'T'T$; $T'T'R'R'T$; $R'T'R'T'T$; $T'R'T'R'T$; $R'T'T'R'T$; $T'R'R'T'T$. Fig. 9 shows these six designs.
(iii) For the group $G2/G3'$ the tool axes are TT so $N_T'=1$, $N_R'=2$, $N_T=2$, $N_R=0$ and Equation (2) gives $N_{COMB}=3$. The three design combinations are: $R'R'T'TT$; $R'T'R'TT$ and $T'R'R'TT$. The group $G2/G3'$ contains three instances of the $R'R'$ machine. These instances are represented in Fig. 10.
(vi) If the tool axes are TTT the workpiece carrying axes can only be $R'R'$. So only one design combination is possible.

From the above-mentioned findings it can also be concluded that the total number of $R'R'$ five-axis machine configurations is 20.

Machines with two axes on the clamping table can be seen in Figs. 1, 3, 6 and 8. The advantages are:

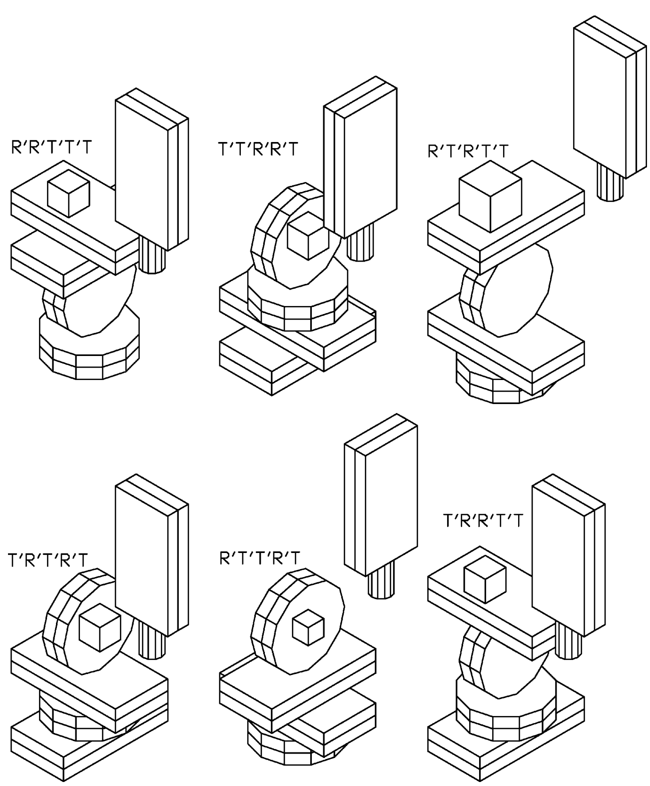

Fig. 9. Members of group $G1/G4'$.

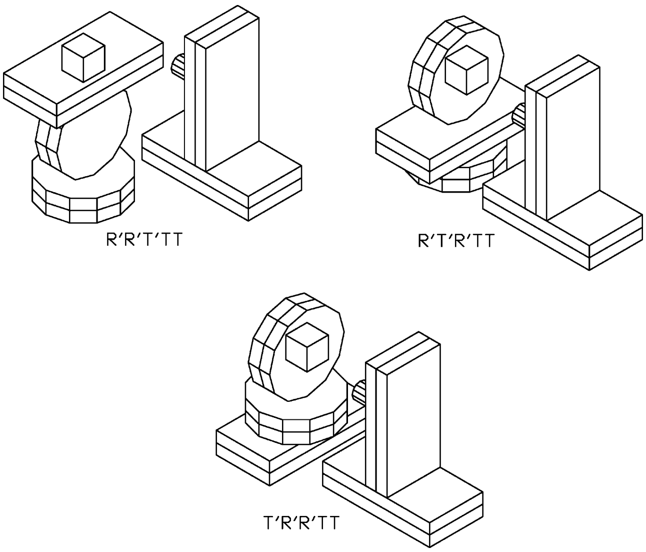

Fig. 10. $R'R'$ machines in group $G2/G3'$.

- In case the spindle is horizontal, optimal chip removal is obtained through the gravitational effect of the chips just dropping.
- The tool axis during machining is always parallel to the *Z* axis of the machine. So the drilling cycles can be executed along the *Z*-axis of the machine. Circles under a certain orientation of the workpiece are always executed in the *XY* plane of the machine. The above-mentioned functions can be executed in the simple three-axis numerical control mode.
- The compensation of the tool length happens all the time in the NC control of the machine, as with three-axis machines.

Disadvantages:

- Machines with a rotating table are only for workpieces with limited dimensions.
- The useful workspace is usually much smaller than the product of the travel in *X*,*Y* and *Z* axis.
- The transformation of the Cartesian CAD/CAM coordinate (*XYZIJK*) of the tool position to the machine axes positions (*XYZAB* or *C*) is dependent on the position of the workpiece on the machine table. This means that in case the position of the workpiece on the table is changed this cannot be modified by a translation of the axes system in the NC program. They must be recalculated. In case the control of the NC machine cannot transform Cartesian coordinates to machine coordinates, then a new CNC program must be generated with the postprocessor of the CAD/CAM system every time the position of the workpiece changes.

Important applications for this type:

- Five-sided cutting of electrodes for EDM and other workpiece.
- Machining of precision workpieces.
- Turbines and tire profiles with a certain workpiece geometry rotated over a certain angle. The same NC program can be repeated after the zero of the rotation axis has been inclined over a certain angle.

4.2.2. RR-machine

The number of possible design combinations (N_{COMP}=20) is the same as in the case of the $R'R'$ machine because of the symmetry. Five-axis machines with the rotation axes implemented on the tool axis spindle can be seen in Figs. 4 and 5.

Advantages:

- These machines can machine very large workpieces.
- The machine axis values of the NC program *XYZ*, depend on the tool length only. A new clamping position of the workpiece is corrected with a simple translation. This happens with a zero translation in the CNC control of the machine.

Disadvantages:

- The drive of the main spindle is very complex. Simple design and construction is only obtained when the whole spindle with the motor itself is rotating.
- There is a lower stiffness because the rotation axis of the spindle is limiting the force transmission. At high revolutions per minute (higher than 5000 rpm) there is also a counter acting moment because of the gyroscopic effect which could be a disadvantage in case the tool spindle is turning very fast.
- Circular interpolation in a random plane and drilling cycles under random orientation are often not implemented.
- A change in the tool length cannot be adjusted by a zero translation in the control unit, often a complete recalculation of the program (or postprocessing) is required.

Important applications of this type of machine tool are:

- All types of very large workpieces such as air plane wings.

4.2.3. $R'R$ machine

One rotary axis is implemented in the workpiece kinematic chain and the other rotary axes in the tool kinematic chain (e.g. Fig. 7).

The groups $G4/G1'$, $G4'/G1$, $G3'/G2$, $G3/G2'$ cover this design. Nowadays there are many machines on the market with one rotation axis on the tool spindle and one rotation axis on the table. They are, however, combining most of the disadvantages of both previous types of machines and are often used for the production of smaller workpieces. The application range of this machine is about the same as with machines with two rotation axes implemented on the table.

In all possible designs of this machine the $N_R'=N_R=1$ and $N_T'+N_T=3$. The total number of possible designs is:

$$N_{\mathrm{COMB}}[N_T=0,N_T'=3]+N_{\mathrm{COMB}}[N_T=1,N_T'=2]+N_{\mathrm{COMB}}[N_T=2,N_T'=1]+N_{\mathrm{COMB}}[NT=3,\ NT'=0]$$

or 4+6+6+4=20 possible designs.

(i) For $N_T'=0$ and $N_T=3$ the four combinations are: *R′RTTT*; *R′TRTT*; *R′TTRT*; *R′TTTR*.
(ii) For $N_T'=1$ and $N_T=2$ the six combinations are: *T′R′RTT*; *T′R′TRT*; *T′R′TTR*; *R′T′RTT*; *R′T′TRT*; *R′T′TTR*.
(iii) For $N_T'=2$ and $N_T=1$ the six combinations are (see Fig. 11): *R′T′T′TR*; *T′R′T′TR*; *T′T′R′TR*; *R′T′T′RT*; *T′R′T′RT*; *T′T′R′RT*.
(iv) For $N_T'=3$ and $N_T=0$ the four combinations are: *R′T′T′T′R*; *T′R′T′T′R*; *T′T′R′T′R*; *T′T′T′R′R*.

5. Workspace of a five-axis machine

Before defining the workspace of the five-axis machine tool, it is appropriate to define the workspace of the tool and the workspace of the workpiece. The workspace of the tool is the space obtained by sweeping the tool reference point (e.g. tool tip) along the path of the tool carrying axes. The workspace of the workpiece carrying axes is defined in the same way (the center of the machine table can be chosen as reference point). These workspaces can be determined by computing the swept volume [6].

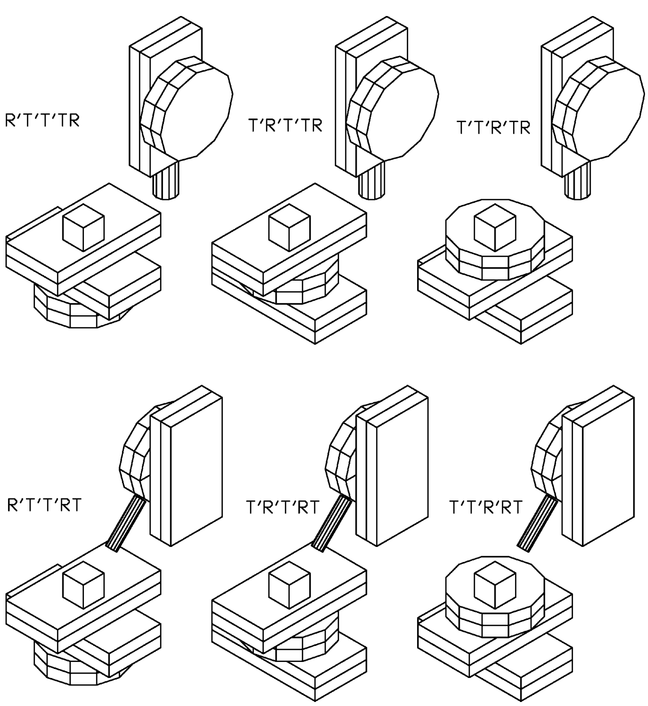

Fig. 11. *R′R* machines in the group *G*2/*G*3′.

Based on the above-definitions some quantitative parameters can be defined which are useful for comparison, selection and design of different types of machines.

5.1. Workspace utilization factor W_R

A possible definition for this is the ratio of the Boolean intersection of the workpiece workspace and tool workspace and the union of the tool workspace and workpiece workspace.

$$W_R=\frac{WS_{TOOL}\cap WS_{WORKPIECE}}{WS_{TOOL}\cup WS_{WORKPIECE}} \quad (4)$$

A large value for W_R means that the workspace of the tool and the workspace of the workpiece are about equal in size and overlap almost completely. A small value of W_R means that the overlap of tool workspace and workpiece workspace is small and that a large part of the workpiece workspace cannot be reached by the tool. The analogy with two cooperating robots can be clearly seen. It is only in the intersection of the two workspaces of each robot that they can ‘shake hands’. For the five-axis machine tool this corresponds to the volume in which the tool and workpiece reference point can meet.

However, in the case where all the five axes carry the workpiece and the tool is fixed in space the above definition would give a zero value for the workspace utilization. In the case of cooperating robots it would mean that there is only one point were they can shake hands. In the case of a five-axis machine, the workpiece can still be moved in front of the tool and remove metal. The reason is that many points from the workpiece can serve as reference point on the workpiece. All points which can cut on the toolsurface can be used as tool reference point. It is therefore necessary to modify the above definition for the case of a five-axis machine. All points of the largest possible workpiece which can be brought into contact with all the tool reference points should be considered as the intersection of the tool workspace and workpiece workspace in the case of five-axis machines. The set of workpiece reference points which can be brought in contact with the set of tool reference points is defined as the machine tool workspace.

$$\mathrm{WS_{MT}}=\cup(\mathrm{WS_{TOOL}}\cap\mathrm{WS_{WORK}}),\forall\mathrm{Tool_{ref}},\forall\mathrm{Work_{ref}}$$

So the above formula should be modified as follows.

$$W_R=\frac{\cup(WS_{TOOL}\cap WS_{WORKPIECE})\forall Tool_{ref},\forall Work_{ref}}{WS_{TOOL}\cup WS_{WORKPIECE}} \quad (5)$$

The union of all possible intersections of the tool workspace and workpiece workspace for all possible tool reference point and workpiece reference points should be used for the numerator.

For the denominator the tool workspace and workpiece workspace are taken for a fixed single reference point. For the tool reference point the center of the spindle nose reference surface is taken. For the workpiece workspace the center of the machine table surface is taken as the reference point.

To each possible meeting point corresponds an orientation of the toolvector and the vector perpendicular to the machine table. In fact for each possible meeting point of the tool and workpiece reference point there can be more then one possible orientation of the tool and machine table.

5.2. Machinable volume

Once the workpiece has been fixed relative to the workpiece reference point, and a specific tool relative to the tool reference point, it is possible to determine the machinable volume. The machinable volume is the total volume which can be removed from the workpiece for a particular instance of the tool and workpiece, and set up of tool and workpiece. The intersection of the machine tool workspace (Section 5.1) and the workpiece give the amount of material which can be removed or the machinable volume, for a particular workpiece and tool setup.

5.3. Machine tool space efficiency MT_S

The machine tool space efficiency is defined as the ratio of the Machine Tool workspace (clipped) and the smallest convex volume which envelopes the machine.

$$MT_S = \frac{\cup(WS_{TOOL} \cap WS_{WORKPIECE}) \bigvee Tool_{ref}, \bigvee Work_{ref}}{Volume} \quad (6)$$

5.4. Orientation space index of a five axis machine OS_I

A way to asses the maximum range of orientation is to determine the largest part of a sphere which can be produced on the machine by using the two rotary axes.

The orientation index is defined as the ratio of the volume of the largest spherical dome which can be machined with the machine using the full range of the rotary axes. Divided by the machine tool workspace.

$$OS_1 = \frac{VOL_{DOME}(R_1,R_2)}{\cup(WS_{TOOL} \cap WS_{WORKPIECE}) \bigvee Tool_{ref}, \bigvee Work_{ref}} \quad (7)$$

If this index is close to unity it means that the full range of the rotary axes can be used in the whole machine tool workspace. If this index is much smaller than unity it means that about OS_I percent of the workspace can use the full range of the rotary axes.

The above definition is a theoretical definition. The real orientation workspace index will be further limited by the need to avoid collision between parts of the machine, tool and workpiece. This will be reflected in a smaller spherical dome which can be machined.

5.5. Outline of algorithm to compute the machine tool workspace

The output of the CAM module is an ordered set of points and vectors: $x,y,z,\mathbf{i},\mathbf{j},\mathbf{k}$. The points are the x,y,z coordinates of the Cutter Location point (or tool reference point) on the tool and the vector components of the tool vector $\mathbf{i},\mathbf{j},\mathbf{k}$. These vectors are given in the workpiece coordinate systems fixed rigidly to the workpiece.

The transformation from workpiece to machine coordinates is called the geometry transform [2] or inverse kinematics. In the case of the five-axis milling machine it is necessary to transform this $x,y,z,\mathbf{i},\mathbf{j},\mathbf{k}$ data to the machine slide position coordinates (further called T_1, T_2, T_3, R_1 and R_2) which control the motions of the machine. In nearly all cases the required motions are obtained by a combined movement of the workpiece and the tool on the specific machine tool. The geometry transformation will transform from the workpiece coordinate system to a preset machine coordinate system fixed to the machine frame.

The workpiece coordinates $x,y,z,\mathbf{i},\mathbf{j},\mathbf{k}$ can be expressed in function of the machine coordinates

$$T_1=F_{T1}[x,y,z,\mathbf{i},\mathbf{j},\mathbf{k}];\ T_2=F_{T2}[x,y,z,\mathbf{i},\mathbf{j},\mathbf{k}];\ T_3=F_{T3}[x,y,z,\mathbf{i},\mathbf{j},\mathbf{k}];\ R_1=F_{R1}[\mathbf{i},\mathbf{j},\mathbf{k}];\ R_2=F_{R2}[\mathbf{i},\mathbf{j},\mathbf{k}] \quad (8)$$

The inverse of this transformations gives:

$$x=F_X[T_1,T_2,T_3,R_1,R_2];\ y=F_Y[T_1,T_2,T_3,R_1,R_2];\ z=F_Z[T_1,T_2,T_3,R_1,R_2];\ \mathbf{i}=F_i[R_1,R_2];\ \mathbf{j}=F_j[R_1,R_2];\ \mathbf{k}=F_k[R_1,R_2] \quad (9)$$

The range of the machine axes is limited by the maximum travel of T_1, T_2, T_3, R_1 and R_2. The limits are the corner points of a five-dimensional hypercube or parallelotope with N_r r-dimensional

$$N_r=2^{n-r}\binom{n}{R} \quad (10)$$

hyper faces, faces, edges and points [12]. To find an approximation of the machine tool workspace a regular five-dimensional pointgrid of equally spaced points can be transformed by putting the coordinates of each point in Eq. (9). The number of points to be transformed is L^5 with L the linear density per axis (e.g. L=10 requires 100,000 transformations). If only the points on the two-dimensional faces of the hypercube are transformed, then only L^2 N_2 points need to be transformed (L=10 gives 8000 points to be transformed). The volume of this point set is the machine tool workspace. If this machine tool workspace is now put on the machine table, then the part which interferes with the machine frame must be subtracted (clipped).

5.6. Outline of algorithm to compute the orientation space index

The equation of a sphere surface is given in spherical coordinates as: $x=r \sin \theta \cos \psi$; $y=r \sin \theta \sin \psi$; $z=r \cos \theta$; the range of ψ is from 0 to 2π and θ from 0 to π can be matched to the range of the machine rotary axes R_1 and R_2. At tool path to mill the largest possible part of a sphere can now be determined. The first step in the determination is the selection of the orientation of the tool axis relative to the sphere. At each point x,y,z of the sphere there are three possible orientations along the unit vectors $\mathbf{i}_r$ $\mathbf{i}_\theta$ and $\mathbf{i}_\psi$. If the tool axis is oriented along $\mathbf{i}_r$ the tool axis will be perpendicular to the sphere surface. If the tool axis is oriented along $\mathbf{i}_\theta$ or $\mathbf{i}_\psi$, the tool axis will be parallel to the isoparametric lines on the sphere θ = constant or ψ= constant. The tool path can be generated with the following equations:

$$x=r \sin(\psi/2n) \cos \psi;\ y=r \sin(\psi/2n) \sin \psi;\ z=r \cos(\psi/n)$$

for r=constant, n=number of turns and $\theta \le \psi \le 2n\pi$.

The CL point will follow a 'spiral' motion with n the number of full turns.

The orientations of the tool vector for each CL point should be selected from the three possible orientation-s:Perpendicular to sphere:

$$\mathbf{i}=\sin(\psi/2n) \cos \psi,\ \mathbf{j}=\sin(\psi/2n) \sin \psi,\ \mathbf{k}=\cos(\psi/2n)$$

Parallel to the isoparameric line θ = constant:

$$\mathbf{i}=\sin(\psi/2n) \cos \psi,\ \mathbf{j}=\sin(\psi/2n) \sin \psi,\ \mathbf{k}=-\sin(\psi 2/n)$$

Parallel to isoparameric line ψ = constant, which is always parallel to the xy plane:

$$\mathbf{i}=-\sin \psi,\ \mathbf{j}=\cos \psi,\ \mathbf{k}=0.$$

Normally it will be clear which toolorientation vector will give the largest r. The number of turns is not important as this toolpath will probably never be cut on the machine so a value, e.g. 10 can be taken. The angle ψ can be related to maximum range of R_1 and R_2 through the geometry transform of the machine. The maximum radius of the sphere can be determined by starting with an initial guess r_0 and check if this is within the range of the machine. The problem can be somewhat be more complicated due to the many possible locations of the sphere on the machine table, here also a good initial guess starts the iteration process. A point x,y,z located on the sphere surface will have a radius $r^2=(x-x_0)^2+(y-y_0)^2+(z-z_0)^2$. With x_0, y_0, z_0 the center of the sphere. The equation to be checked is:

$$r^2 \le F_X[T_1,T_2,T_3,R_1,R_2]^2+F_Y[T_1,T_2,T_3,R_1,R_2]^2 +F_Z[T_1,T_2,T_3,R_1,R_2]^2$$

This relation must hold for all corner points of the five-dimensional parallelotope which defines the machine range. Using equation (10) gives $N1$=80 corner points.

5.7. Orientation angle index of a five-axis machine OA_I

If a large range of orientation is required it should be possible to make a complete sphere. To make a complete sphere two RR axis are needed. One axis with a range of 360° and another perpendicular axis with a range of 180° are the minimum requirement. The Orientation angle index is defined as the ratio of the product of the max range of the two rotary axes divided by 360×180 multiplied by α_{12}/90. With α_{12} the angle in degrees between the two rotary axes (e.g. in Fig. 6, α_{12}=45°).

$$OA_I=\frac{\Delta R_1 \cdot \Delta R_2}{360° \cdot 180°}\frac{\alpha_{12}°}{90°} \tag{11}$$

If this index is 1 it is possible to mill a full sphere. Often there is only a subspace of the workspace available in which this full range of the two rotary axes orientations can be used. To be able to asses the size of the part OA_I index should be used in combination with OS_I

6. Selection criteria of a five-axis machine

It is not the objective to make a complete study on how to select or design a five-axis machine for a certain application. Only the main criteria which can be used to justify the selection of a five-axis machine are discussed.

6.1. Applications of five-axis machine tools

The applications can be classified in positioning and contouring. Figs. 12 and 13 explain the difference between five-axis positioning and five-axis contouring.

6.1.1. Five-axis positioning

Fig. 12 shows a part with a lot of holes and flat planes under different angles, to make this part with a three-axis milling machine it is not possible to process the part in one set up. If a five-axis machine is used the tool can

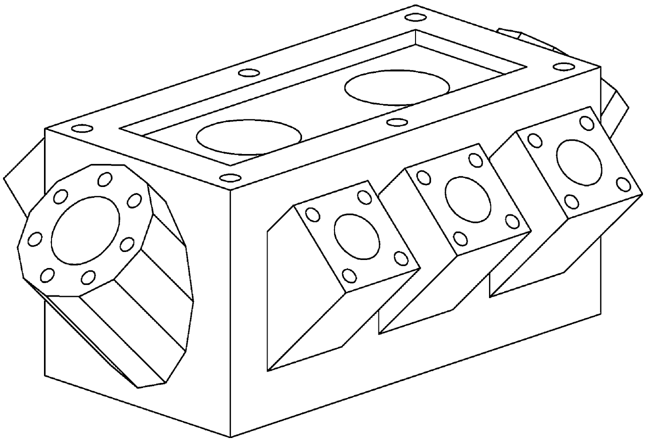

Fig. 12. Five-axis positioning.

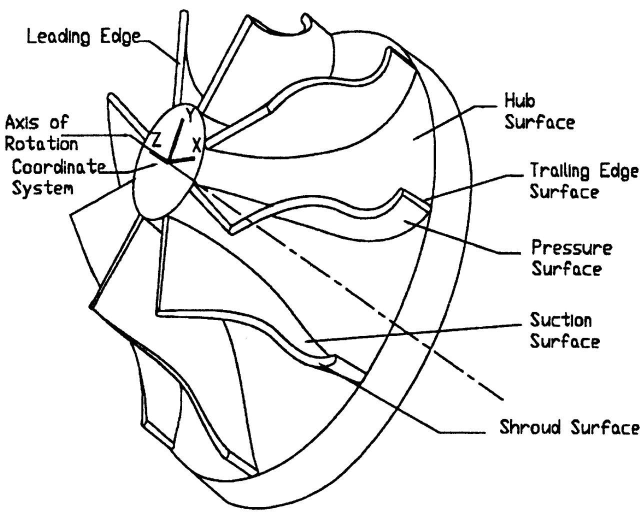

Fig. 13. Five-axis contouring.

be oriented relative to the workpiece in any direction. Once the correct position is reached, the holes or the flat planes can be machined while keeping most of the axis positions fixed. The flat planes can also include 2D pockets with islands. If only holes need to be drilled it is in theory sufficient to have a one axis simultaneous CNC control, with 2D pockets a 2 axis simultaneous CNC control is sufficient. However, simultaneous control of three axes is now common. This increases the speed in the rapid traverse mode when the tool and workpiece are positioned relative to each other before any cutting takes place.

6.1.2. Five-axis contouring

Fig. 13 shows an example of five-axis contouring, to machine the complex shape of the surface we need to control the orientation of the tool relative to the part during cutting. The tool workpiece orientation changes in each step. The CNC controller needs to control all the five-axes simultaneously during the material removal process. More details on countouring can be found in Ref. [13]. Applications of five-axis contouring are: (i) production of blades, such as compressor and turbine blades; (ii) injectors of fuel pumps; (iii) profiles of tires; (iv) medical prosthesis such as artificial heart valves; (v) molds made of complex surfaces.

6.2. Axes configuration selection

The size and weight of the part is very important as a first criterion to design or select a configuration. Very heavy workpieces require short workpiece kinematic chains. Also there is a preference for horizontal machine tables which makes it more convenient to fix and handle the workpiece. Putting a heavy workpiece on a single rotary axis kinematic chain will increase the orientation flexibility very much. It can be observed from Fig. 4 that providing a single horizontal rotary axis to carry the workpiece will make the machine more flexible.

In most cases the tool carrying kinematic chains will be kept as short as possible because the toolspindle drive must also be carried.

6.3. Example 1 — five-axes machining of jewelry

A typical workpiece could be a flower shaped part as in Fig. 14. This application is clearly contouring. The part will be relatively small compared to the tool assembly. Also small diameter tools will require a high speed spindle. A horizontal rotary table would be a very good option as the operator will have a good view of the part (with range 360°). All axes as workpiece carrying axes would be a good choice because the toolspindle could be fixed and made very rigid. There are 20 ways in which the axes can be combined in the workpiece kinematic chain (Section 4.2.1). Here only two kinematic chains will be considered. Case one will be a $T'T'T'R'R'$ kinematic chain shown in Fig. 15. Case two will be a $R'R'T'T'T'$ kinematic chain shown in Fig. 16.

For model I a machine with a range of X=300 mm,

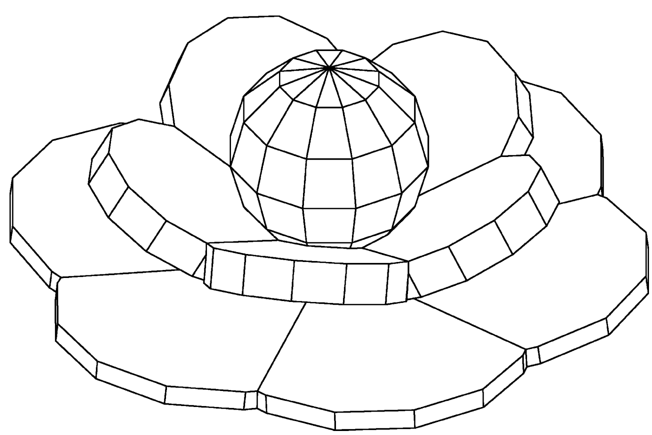

Fig. 14. Jewel application

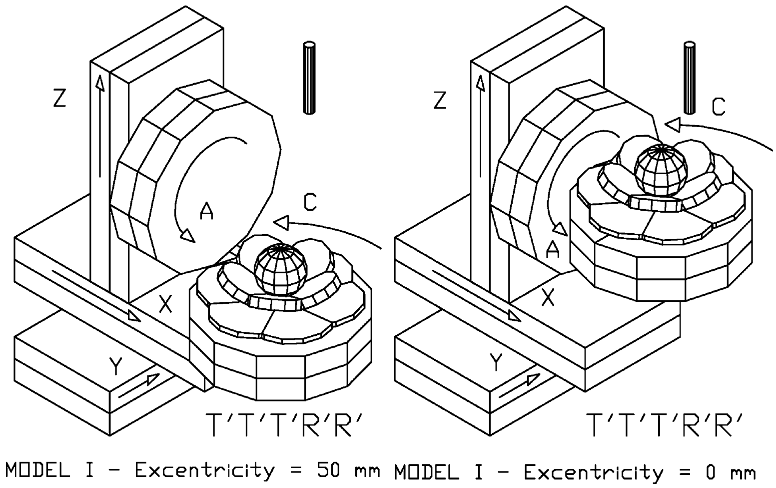

Fig. 15. $T'T'T'R'R'$ five-axis machine.

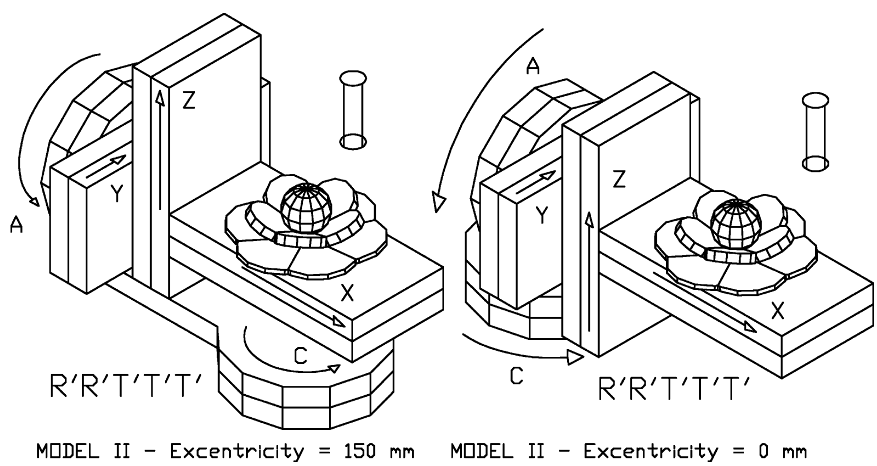

Fig. 16. $R'R'T'T'T'$ five-axis machine.

Y=250 mm, Z=200 mm, C=n 360° and A=360°, and a machine tool table of 100 mm diameter will be considered. For this kinematic chain the tool workspace is a single point. The set of tool reference points which can be selected is also small. With the above machine travel ranges the workpiece workspace will be the space swept by the center of the machine table. If the centerline of the two rotary axes intersect in the reference point, a prismatic workpiece workspace will be obtained with as size XYZ or 300×250×200 mm^3. If the centerlines of the two rotary axes do not intersect in the workpiece reference point then the workpiece workspace will be larger. It will be a prismatic shape with rounded edges. The radius of this rounded edge is the excentricity of the workpiece reference point relative to each centerline.

Model II in Fig. 15 has the rotary axes at the beginning of the kinematic chain ($R'R'T'T'T'$). Here also two different values of the rotary axes excentricity will be considered. The same range of the axes as in model I is considered.

The parameters defined in Section 5 are computed for each model and excentricity and summarized in Table 1. It can be seen that with the rotary axes at the end of the kinematic chain (model I), a much smaller machine tool workspace is obtained. There are two main reasons for this. The swept volume of the tool and workpiece $WS_{TOOL} \cup WS_{WORK}$ is much smaller for model I. The second reason is due to the fact that a large part of the machine tool workspace cannot be used in the case of model I, because of interference with the linear axes. The workspace utilization factor however is larger for the model I with no excentricity because the union of the tool workspace and workpiece workspace is relatively smaller compared with model I with excentricity e=50 mm. The orientation space index is the same for both cases if the table diameter is kept the same. Model II can handle much larger workpieces for the same range of linear axes as in model I. The rotary axes are here in the beginning of the kinematic chain, resulting in a much larger machine tool workspace then for model I. Also there is much less interference of the machine tool workspace with the slides. The other 18 possible kinematic

Table 1
Workspace comparison of two five-axis machines

	$T'T'T'R'R'$	Model I	$R'R'T'T'T'$	Model II
Excentricity	e=0 mm	e=50 mm	e=0 mm	e=150 mm
WS_{MT}	25.13 dm^3	25.13 dm^3	48.0 dm^3	32.4 dm^3
WS_{MT} clipped	14.57 dm^3	14.57 dm^3	48.0 dm^3	32.4 dm^3
$WS_{TOOL} \cup WS_{WORK}$	15 dm^3	30.85 dm^3	107.7 dm^3	39.8 dm^3
W_R	0.97	0.47	0.44	0.814
ϕLargest sphere	ϕ100 mm	ϕ100 mm	ϕ300 mm	ϕ250 mm
OS_I	0.036	0.036	0.29	0.25
OA_I	2	2	2	2
Space	32 dm^3	45 dm^3	100.53 dm^3	56.55 dm^3
MT_S	0.47	0.57	0.48	0.57

chain selections will give index values somewhat in between the above cases.

6.4. Example 2 — rotary table selection

Two machines with the same kinematic diagram ($T'T'R'R'T$) and the same range of travel in the linear axes will be compared (Fig. 17).

There are two options for the rotary axes: two-axis table with vertical table (model I), two-axis table with horizontal table (model II). Tables 2 and 3 give the comparison of the important features.

It can be observed that reducing the range of the rotary axes increases the machine tool workspace. So model I will be more suited for smaller workpieces with operations which require a large orientation range, typically contouring applications. Model II will be suited for larger workpieces with less variation in tool orientation or will require two setups. This extra setup requirement could be of less importance then the larger size. The horizontal table can use pallets which transform the internal setup to external setup.

The larger angle range in the B-axes −105 to +105, compared to −45 to +20, makes model I more suited for complex sculptured surfaces, also because the much higher angular speed range of the vertical angular table. The option with the highest spindle speed should be selected and it will permit the use of smaller cutter diameters resulting in less undercut and smaller cutting forces. The high spindle speed will make the cutting of copper electrodes for die sinking EDM machines easier. The vertical table is also better for the chip removal. The large range of angular orientation, however, reduces the maximum size of the workpiece to about 300 mm and 100 kg. Model II with the same linear axes range as model I, but much smaller range in the rotation, can easily handle a workpiece of double size and weight. Model II will be good for positioning applications. Model I cannot be provided with automatic workpiece exchange, making it less suitable for mass production. Model II has automatic workpiece exchange and is suitable for mass production of position applications.

Model I could, however, be selected for positioning applications for parts such as hydraulic valve housings which are small and would require a large angular range.

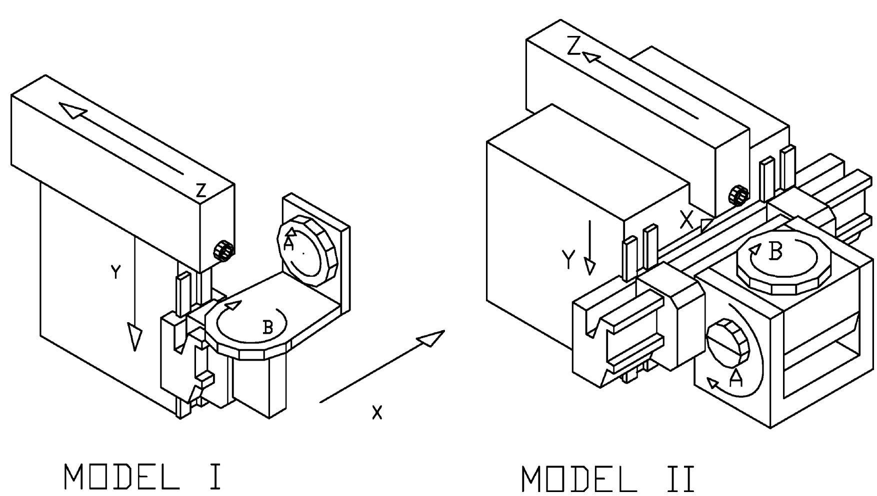

Fig. 17. Model I and model II $T'T'R'R'T$ machines.

Table 2
Specifications of five-axis machine tools in Fig. 17

Machine type	Model I	Model II
Range of *B*-axis	−105 to +105°	−45 to +20°
Range of *A*-axis	$n\times360°$	$n\times360°$
Angular speed °/s	8500/14,000	3000/3000
Type of table	Vertical	Horizontal
Diameter table	320 mm	520 mm
Max. table load	100 kg	200 kg
X Y Z range	600×450×500 mm	600×450×500 mm
Toolnose holding	ISO 40	ISO 40
Weight	4500 kg	5500 kg
Pallet exchange	No	Yes

Table 3
Workspace comparison of five-axis machine tools in Fig. 17

	Model I	Model II
WS_{MT}	212.0 dm^3	145.6 dm^3
WS_{MT} clipped	84.0 dm^3	145.6 dm^3
$WS_{TOOL}\cup WS_{WORK}$	334.5 dm^3	180.5 dm^3
W_R	0.25	0.81
ϕLargest sphere	ϕ300 mm	ϕ930 mm
OS_I	0.17	0.05
OA_I	1.17	0.36
Space	9000 dm^3	9000 dm^3
MT_S	0.037	0.07

6.5. Selection of machine options

6.5.1. Automatic tool change (ATC)

An automatic tool change on a CNC machine is a very useful option for mass production. The most important feature to look for when selecting this option is the tool exchange time. Today machines with a tool exchange time below 1 s are available. An ATC should, however, not be a high priority option for a mold and die making shop. It can however be very useful if unattended machining is required. An ATC on a machine tool increases the downtime of the machine. Frequent breakdown is due to two reasons [14]:

- Breakdown of the hydraulic or pneumatic system.
- When power fails and the tool exchange arm is halfway the toolexchange operation, it can happen that the arm has to be withdrawn by manually actuating the control valves in the correct sequence.

6.5.2. Automatic pallet change (APC)

This option is again very useful for small to large series production. A requirement for APC is that the machine table which holds the workpiece is horizontal. Many machines equipped with APC have two or more tables in carrousel which allows the set up (still a difficult job to automate) of the part while the machine is working on another part on the machine.

The configuration of Fig. 1 provides the machine tool designer with an easy design for the tool exchanger, the provision of an automatic workpiece exchange is, however, much more difficult and normally not provided, because of the vertical table. The configuration of Fig. 3 allows both Automatic Tool Exchange (ATC) and Automatic Workpiece Exchange (AWC).

6.5.3. Horizontal or vertical spindle machining center

The most popular machining center for series production is the horizontal machining center (tool spindle horizontal — Fig. 18). The main reasons for this are [15]:

- Chips drop out of the way during machining, providing an uncluttered view of the cut and preventing recutting of chips.
- The table indexing capability enables multiple sides of a workpiece to be machined in one setup.
- Easy to provide APC.

The disadvantages of the horizontal machining center are:

- Heavy tools deflect.

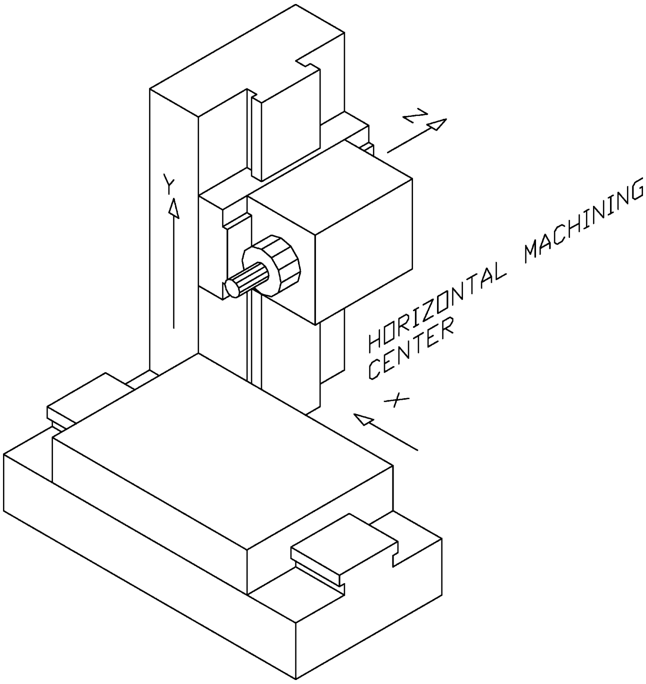

Fig. 18. Horizontal spindle machine.

- Trust of cutting tool must be absorbed by fixtures.
- Generally more expensive than the vertical spindle machine.

The most popular machine for a job shop and moldmaker is the vertical spindle machine (Fig. 19). The main reasons are:

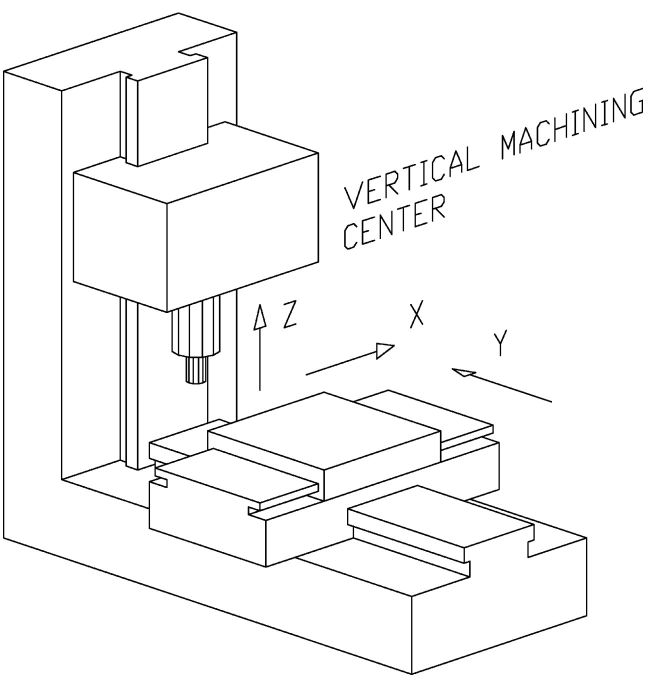

Fig. 19. Vertical spindle machine.

- Trust of the cutting tool is directly absorbed into the machine table.
- Ideal for large, flat plate work and single surface 3D contouring.
- Heavy tools can be used without concern about deflection.
- Generally less costly.

The main disadvantage is the extensive chip buildup which obstructs the view and recuts chips

7. New machine concepts based on the Stewart platform

Conventional machine tool structures are based on Carthesian coordinates. Many surface contouring applications can be machined in optimal conditions only with five-axis machines. This five-axis machine structure requires two additional rotary axes. To make accurate machines, with the required stiffness, able to carry large workpieces, very heavy and large machines are required. As can be seen from the kinematic chain diagram of the classical five-axis machine design the first axis in the chain carries all the subsequent axes. So the dynamic responce will be limited by the combined inertia. A mechanism which can move the workpiece without having to carry the other axes would be the ideal. A new design concept is the use of a 'HEXAPOD'. Stewart [16] described the hexapod principle in 1965. It was first constructed by Gough and Whitehall [20] in 1954 and served as tire tester. Many possible uses were proposed but it was only applied to flight simulator platforms. The reason was the complexity of the control of the six actuators. Recently with the amazing increase of speed and reduction in cost of computing, the Stewart platform is used by two American Companies in the design of new machine tools. The first machine is the VARIAX machine from the company Giddings and Lewis, USA. The second machine is the HEXAPOD from the Ingersoll company, USA. The systematic design of Hexapods and other similar systems is discussed in Ref. [17].

The problem of defining and determining the workspace of virtual axis machine tools is discussed in Ref. [18]. It can be observed from the design of the machine that once the position of the tool carrying plane is determined uniquely by the CL date (point + vector), it is still possible to rotate the tool carrying platform around the tool axis. This results in a large number of possible length combinations of the telescopic actuators for the same CL data.

7.1. Variax machine

This machine approaches the Stewart platform very closely. One upper and lower platform is connected by six crossed telescopic linear axes (Fig. 20). These telescopic axes can expand and retract by means of revolving ballscrew spindles and a servomotor per axis. The bottom platform caries the workpiece pallet and the top platform the toolpallet. The top platform carries the machining spindle and drive. The lightweight triangular framework provides high stiffness. The telescopic actuators are supported at the two endpoints. This results in only compressive or tension load resulting in high stiffness. Further variations on the same structure are:

- The machine is made with a vertical platform for carrying the workpiece, getting the benefits of a horizontal spindle machine.
- The duplicating tool platform on both sides of the workpiece platform would increase the productivity by reducing the number of setups and two tools cutting simultaneously.

7.2. Hexapod machine

The HEXAPOD (Fig. 21) machine does not resemble the Stewart platform as closely as the VARIAX machine. To avoid the large space taken by the six telescopic axes in the VARIAX machine, the six axes are connected to the (fixed) top platform instead of the bottom. To be able to realize this construction it was necessary to have a machine mainframe to support this upper fixed platform. This frame is also made from a triangular beam mesh. The movable platform is now much smaller and carries the spindle and its drive. This machine has a much larger useful workspace than the VARIAX machine. The telescopic axes are larger. The tool carrying platform is much smaller and the mass to be moved is smaller.

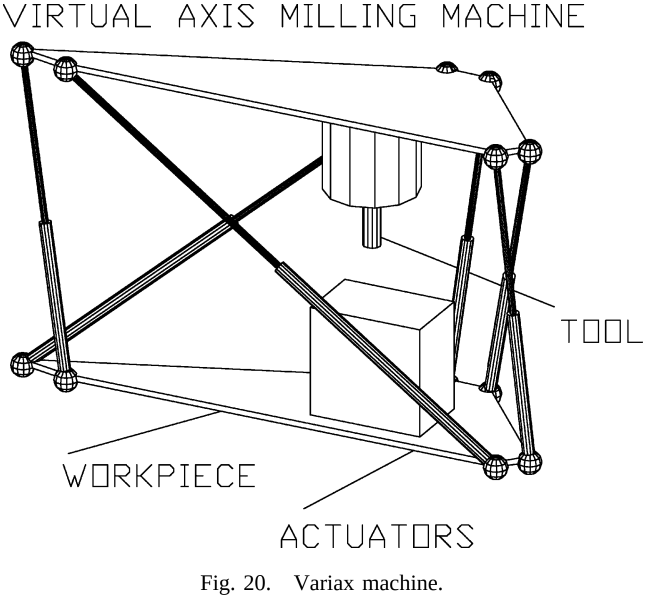

Fig. 20. Variax machine.

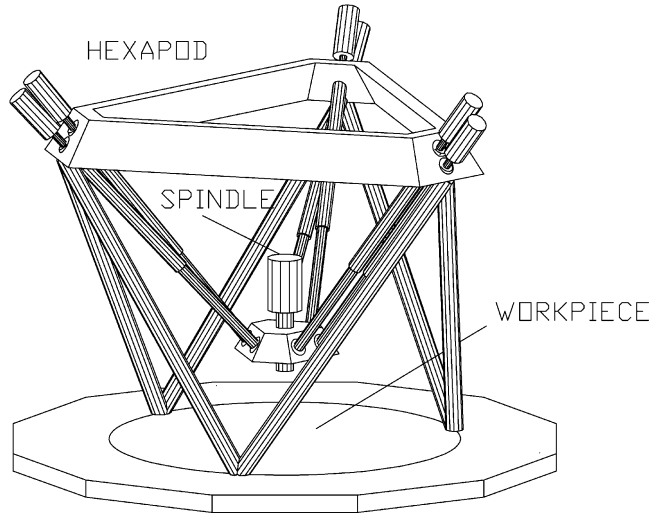

Fig. 21. Hexapod machine.

Advantages:

- Stiff construction.
- The telescopic legs are only loaded in compression and tension.
- Simple assembly.
- All leg drives are the same (repetitive construction).
- Small moving masses.
- No special fabrication or assembly features are required for the machine elements.

Disadvantages:

- A six axis CNC control is required.
- The build in coordinate transform is a heavy load for the CNC control.
- The tilting angle is limited to ±15 degrees. So if full five-axis capability is required, a tilting and rotary table will be necessary. So that the machine is not really a six-axis machine.
- Large thermal expansions.
- Unfavorable workspace to machine volume ratio.

8. Conclusion

Theoretically there are large number of ways in which a five-axis machine can be built. Nearly all classical Cartesian five-axis machines belong to the group with three linear and two rotational axes or three rotational axes and two linear axes. This group can be subdivided in six subgroups each with 720 instances. If only the instances with three linear axes are considered there are still 360 instances in each group. The instances are differentiated based on the order of the axes in both tool and workpiece carrying kinematic chain.

If only the location of the rotary axes in the tool and workpiece kinematic chain is considered for grouping five-axis machines with three linear axes and two rotational axes, three groups can be distinguished. In the first group the two rotary axes are implemented in the workpiece kinematic chain. In the second group the two rotary axes are implemented in the tool kinematic chain. In the third group there is one rotary axis in each kinematic chain. Each group still has twenty possible instances. To determine the best instance for a specific application area is a complex issue. To facilitate this some indexes for comparison have been defined such as the machine tool workspace, workspace utilization factor, orientation space index, orientation angle index and machine tool space efficiency. An algorithm to compute the machine tool workspace and the diameter of the largest spherical dome which can be machined on the machine was outlined. The use of these indexes for two examples was discussed in detail. The first example considers the design of a five-axis machine for jewelry machining. The second example illustrates the selection of the rotary axes options in the case of a machine with the same range in linear axes.

The most widely used five-axis machines have the two rotary axes implemented on the workpiece side at the end of the kinematic chain. This construction provides a modular design for the machine tool builder. This modular design is, however, not always optimal from the application point of view. Because of the large number of theoretically possible configurations, it is clear that a specific five-axis machine will be most appropriate for a special set of workpieces. Modular designs should be based on modularity in the combination of all the five axes. Current modularity in design is based on a three linear axis machine.

Five-axis milling offers reduction in a number of setups. This helps to increase the accuracy and reduce the lot size. There are, however, some disadvantages: (i) high price of five-axis machines; (ii) additional rotation axes cause additional position error; (iii) the higher cutting speeds on the machine axes for the same feed.

The purchase of a five-axis machine must be preceded by a profound study of the range of products which have to be machined. The parts should be classified either as five-axis positioning or five-axis contouring or both. Machines with a turning table are for example very good to produce rotational workpieces such as compressors. One rotary axis on the tool side and one rotary axis on the workpiece side will provide a larger workspace utilization factor.

The recently introduced virtual axis machines have as a main advantage, the potential of higher dynamical response and higher stiffness. The workspace utilization factor is however much lower in comparison with the classical five-axis machine. The higher rigidity of these machines makes them very appropriate for the design of high speed spindles [19] needed for high speed milling.

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