Revealing of Independent Oscillations in Planetary Reducer Gear owing to its symmetry

L.Banakh* Yu. Fedoseev[+]
Mechanical Engineering Research Institute of Russian Academy of Sciences
Moscow, Russia

Abstract - *The planetary reducer[11] gear is a symmetric system. For its oscillation analysis there is applied the symmetry group representation theory, which was generalized for mechanical systems. It was found that due to reducer symmetry the oscillations decomposition has arisen. There are independent oscillations classes, such as: angular oscillations of solar gear and epicycle - satellites oscillations in phase; transversal oscillations of solar gear and epicycle - satellites oscillations in anti phase. Solar gear and epicycle oscillations in a phase do not depend on angular satellites oscillations.*

Keywords: planetary reducer, symmetry, group representation theory, independent oscillations

I. Introduction

It is well known that at the operation of planetary reducer there are oscillations of its elements, such as solar gear, epicycle and satellites. This factor essentially worsens a quality of reducer' operation, and in some cases can result in their curvature and breakage. A plenty of papers are devoted to the dynamic analysis of gear reducers [1]. Basically there are computational researches. In the given paper the analytical approaches for investigation of reducer dynamics is presented.

The planetary reducer has a high degree of symmetry. So this property was used and the group representation theory was applied. Application of this theory allows carrying out deep enough dynamic analysis, using symmetry properties only. For this purpose it is necessary to have the dynamical model which is taking into account stiffness characteristics in linkages between reducer elements.

The mathematical apparatus of the symmetry groups' representation theory is widely used in the quantum mechanics, crystallographic, spectroscopy [2, 3, 4]. The advantages of this approach are difficult for overestimating. With its help it is possible to define with exhaustive completeness the dynamic properties, using structure symmetry of system only without solving of motion equations. However in the classical mechanics this approach is not widely used. It is result from some particular features of mechanical systems. First, there is an availability of solids with 6-th degrees of freedom. It is unclear to what symmetry group to relate a solid in order that system symmetry may be retained. Second at real designs may be technological errors and mistakes at assembly, so there is a small asymmetry and the system becomes quasi symmetric

Further the mechanical systems consist from various subsystems with various symmetry groups.

In this connection it is necessary to have methods for the analysis as symmetric and quasi symmetric mechanical systems consisting of various subsystems and solids. Having made some generalizations, this mathematical apparatus for mechanical systems may be used. For this purpose we propose to apply the generalized projective operators [5]. These operators are matrixes of the appropriate order instead of scalar as in physics. The use of generalized projective operators allows taking into account all above mentioned features of mechanical systems. The application of these operators to initial stiffness matrix leads to its decomposition on independent blocks each of them corresponds to own oscillation class in independent subspaces. To account for the solids symmetry «the equivalent points» were entered: these points are chosen on solid so that their displacements were compatible to connections and corresponded to group of symmetry of all system.

These operators enable also may be applied with the finite elements models (FEM).

II. Dynamic model of planetary reducer. Stiffness matrix.

The model of a planetary reducer step is submitted on fig. 1 [6]. The step consists from solar gear S, its mass and radius are equal to m_1, r_1. It engages into mesh with three satellites St_i (i=1,2,3) (its masses and radius are identical and equal to m_2, r_2). Satellites in turn are engaged into mesh with epicycle Ep (m_3, r_3) and they are fasten on carrier by elastic support with rigidity h_6.

The rigidity of gearing solar gear-satellites is equal to h_1, the gearing epicycle-satellites is h_3, γ is angle of gearing.

* E-mail: banl@inbox.ru

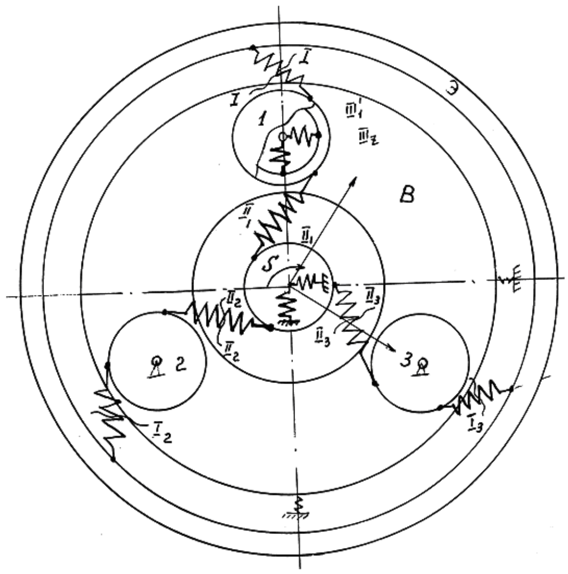

Fig.1 Planetary reducer step.
S- solar gear, Э- epicycle, 1,2,3 –satellites (St).

Let's consider all over again the plane oscillations of planetary reducer step: transversal (*x, y*) and angular (φ) oscillations (without the casing). A stiffness matrix may be represented in a block view

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_S & & \mathbf{K}_{SSt_1} & \mathbf{K}_{SSt_2} & \mathbf{K}_{SSt_3} \\ & \mathbf{K}_Э & \mathbf{K}_{ЭSt_1} & \mathbf{K}_{ЭSt_2} & \mathbf{K}_{ЭSt_3} \\ & & \mathbf{K}_{St_1} & & \\ & & & \mathbf{K}_{St_2} & \\ & & & & \mathbf{K}_{St3} \end{bmatrix} \qquad (1)$$

Here on the main diagonal there are the stiffness submatrixes (3x3) for appropriate elements, and outside of the main diagonal there are stiffness submatrixes of connection between these elements.

There are 15 generalized coordinates:

$$\mathbf{X}=(x_S^*, y_S^*, \varphi_S^*;\ x_{Ep}^*, y_{Ep}^*, \varphi_{Ep}^*, ;\ x_{St_1}, y_{St_1}, \varphi_{St_1} ... \varphi_{St_3})$$

The concrete view of these blocks is submitted in Appendix.

Thus, the total order of matrix **K** is (15x15). An inertia matrix **M** is diagonal.

III. Introduction of equivalent points in dynamic model. Operators of symmetry.

By virtue of symmetry of satellites fastening this subsystem has symmetry such as C_3 (as triangle).

To reveal symmetry C_3 at moving of solar gear S and epicycle Ep we shall enter the coordinates l_1, l_2, l_3 on solar gear **S** in points of satellites fastening (fig. 2.).

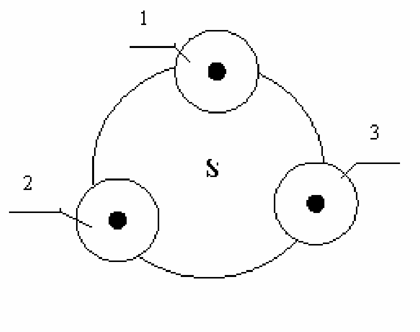

Fig.2 Equivalent points on solar gear **S**. 1,2,3,- satellites

They are "equivalent points". Their coordinates are:

$$\begin{bmatrix} \ell_{1S} \\ \ell_{2S} \\ \ell_{3S} \end{bmatrix} = \begin{bmatrix} \alpha_1 & \beta_1 & r_1 \\ \alpha_2 & \beta_2 & r_1 \\ \alpha_3 & \beta_3 & r_1 \end{bmatrix} \begin{bmatrix} X_S \\ Y_S \\ \varphi_S \end{bmatrix}; \quad \alpha_i = \cos\frac{2\pi(i-1)}{3} \quad \beta_i = \sin\frac{2\pi(i-1)}{3}$$

$$i = 1, 2, 3. \qquad (2)$$

or in matrix form

L=AX

And analogues relations for" equivalent points" on epicycle, but instead r_1 in (2) must be written r_3. And later on these coordinate of solar gear and epicycle will be used instead (x, y) and (φ).

After that it is possible to count, that all coordinates of system should vary according to symmetry group C_3 and, hence, it is possible to apply the projective operator of symmetry to all system elements: S, Ep, and also to three satellites St $_i$ (i =1, 2, 3).(fig.3)

The ortho-normal projective operator g of symmetry for point group C_3 is known as [2]. It is

$$\mathbf{g} = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{-1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & \frac{-1}{\sqrt{6}} \\ \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \qquad (3)$$

For the whole system the projective operator must be represented as block-diagonal matrix

$$\mathbf{G}=\begin{bmatrix} \mathbf{g} & & \\ & \mathbf{g} & \\ & & \mathbf{g}_{St} \end{bmatrix} \qquad (4)$$

Here each sub matrix corresponds to S, Ep, and also to three satellites St $_i$ (i=1, 2, 3). So

Because we have three identical satellites and each of them has 3 degrees of freedom (x_{St_i} , y_{St_i} and angular $\varphi_{St_i} ... \varphi_{St_i}$), therefore it is necessary to enter generalize operator (3) [3,4] and to consider $\mathbf{g}_{St}$ as block matrix where the each element is diagonal matrix (3x3), that is it is possible to present each element as

$$\mathbf{g}_{St} = \mathbf{g}\begin{bmatrix} \mathbf{E} & & \\ & \mathbf{E} & \\ & & \mathbf{E} \end{bmatrix}, \quad \mathbf{E} = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$$

Thus to initial coordinates $(x, y, \varphi)_{S,Ep}$ of solar gear and epicycle consistently two transformations are applied: **A** and **G**. And resulting transformation of an initial matrix **K** equals to product of operators **G A**. This orthogonal transformation and it looks like

$$\mathbf{GA}=\begin{bmatrix} \mathbf{gA} & & \\ & \mathbf{gA} & \\ & & \mathbf{g}_{St} \end{bmatrix}, \text{ where } \mathbf{gA}=\begin{bmatrix} 0 & 0 & 1 \\ \alpha_2 & \beta_2 & 0 \\ +\beta_3 & \alpha_3 & 0 \end{bmatrix}$$

By applying of this transformation to matrix **К** (1), we shall receive

$$\mathbf{К^* = (GA)(К)(GA)^{tr}}$$

So the corresponding transformations of coordinates and forces are

$$\mathbf{X^*=(GA)X}, \qquad \mathbf{F^*=(GA)^{tr}F} \qquad (5)$$

As a result the initial matrix **K** (15x15) is divided on 3 independent blocks (5x5) and, looking like,

$$\mathbf{K}^* = \begin{bmatrix} \mathbf{K}_{\mathbf{I}}^* & & \\ & \mathbf{K}_{\mathbf{II}}^{*(1)} & \\ & & \mathbf{K}_{\mathbf{II}}^{*(2)} \end{bmatrix} \qquad (6)$$

The inertia matrix M remains diagonal because matrix **G A** is orthogonal; therefore the independence of oscillation classes defines matrix **К*** only.

IV. Revealing of independent motions classes at for natural and forced oscillations

A. Natural oscillations

From the view of matrix (6) it is seen, that owing to system symmetry there is a decomposition of initial matrix **K**, and, hence, division of oscillation classes and as well as space of parameters. The concrete relations for submatrixes in (6) show that there are following independent oscillations classes:

I-st class (subspace I- submatrix $\mathbf{K}_{\mathbf{I}}^*$): angular oscillation of solar gear and epicycle + oscillations of satellites in a phase. Dimension of this subspace is equal to 5. Its determining parameters are:

$r_1, r_2, r_3, h_1, h_3, h_6, h_{12}, r_{13}, h_9$.

2-nd class (subspace II- submatrixes $\mathbf{K}_{\mathbf{II}}^{*(1)}$ $\mathbf{K}_{\mathbf{II}}^{*\,(2)}$): transversal oscillations of solar gear and epicycle + oscillations of satellites in an anti phase. Subspace II breaks up to two identical submatrixes $\mathbf{K}_{\mathbf{II}}^{*(1)}$ and $\mathbf{K}_{\mathbf{II}}^{*\,(2)}$ (5x5). It means that in system there are 5 equal frequencies. Its determining parameters are: $r_2, h_1, h_3, h_6, \gamma, h_7, h_9$.

Thus, taking into account only properties of symmetry it is possible to receive deep enough analysis of dynamic properties of system of a planetary reducer. Besides it is possible to simplify also process of system optimization.

B. Forced oscillations

At the forced oscillations the use of the independent oscillation classes is suitable only in two cases: a) if the points of application of the external forces have the same type of symmetry, as a design has, or b) if they are disposed according to the independent classes of oscillations. Really, then transformation (5) bring a forces vector $\mathbf{F}^*$ into a form containing zero elements or in 1-st, or 2-th subspaces.

The analysis of the real loadings forces on a reducer, shows, that it is valid if elements disbalances are the same: a) identical satellites disbalances + disbalance of epicycle; б) identical satellites disbalances + disbalance of solar gear.

V. The further motions decomposition.

The further decomposition of subspaces I and II in (6) is possible only if there are additional conditions raising a type of system symmetry.

These conditions, in particular, can be received from similarity symmetry of solar gear and epicycle. They look like:

1. Equality of gearing stiffness with S and Ep, i.e. $h_1 = h_2$,

2. Equality of partial frequencies for angular motions S and Ep

$\nu_{(\varphi)_S} = \nu_{(\varphi)_{Ep}}$, whence: $h_7 = h_8$,

or 3. Equality of partial frequencies at tranversal motions of S and Ep

$\nu_{(x,y)_S} = \nu_{(x,y)_{Ep}}$, whence: $h_7 = 2h_9$.

So by fulfillment of conditions 1,2 (or 1,3) the additional symmetry type C_{2v} is appeared (reflection symmetry). To this symmetry group the operator $\mathbf{G}_2'$ (or $\mathbf{G}_2''$) is corresponded

$$\mathbf{G}_2' = \begin{bmatrix} \frac{1}{\sqrt{3} r_1 h_1} & \\ & \frac{1}{\sqrt{3} r_3 h_1} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix};$$

$$\mathbf{G}_2'' = \begin{bmatrix} \frac{1}{\sqrt{\frac{3}{2}} h_1} & \\ & \frac{1}{\sqrt{\frac{3}{2}} h_1} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

The application of these operators to matrix $\mathbf{K^*}$ permit to achieve the further decomposition of corresponding matrixes and appropriate motions. Really they may have symmetric and anti symmetric oscillation classes for solar gear and epicycle. Thus the coordinate transformation is:

$$\left.\begin{aligned} \mathbf{X}' &= \frac{1}{\sqrt{3} h_1}\left(\frac{1}{r_1}\mathbf{X}_S^* + \frac{1}{r_3}\mathbf{X}_{Ep}^*\right) \\ \mathbf{X}'' &= \frac{1}{\sqrt{3} h_1}\left(\frac{1}{r_1}\mathbf{X}_S^* - \frac{1}{r_3}\mathbf{X}_{Ep}^*\right) \end{aligned}\right\}$$

And

$$\left.\begin{aligned} \bar{\mathbf{X}}' &= \frac{1}{\sqrt{\frac{3}{2}} h_1}\left(\mathbf{X}_S^* + \mathbf{X}_{Ep}^*\right) \\ \bar{\mathbf{X}}'' &= \frac{1}{\sqrt{\frac{3}{2}} h_1}\left(\mathbf{X}_S^* - \mathbf{X}_{Ep}^*\right) \end{aligned}\right\}$$

By this coordinate transformation the following independent motion types are arisen

$$\mathbf{K}_\mathbf{I}^* \Rightarrow \begin{bmatrix} (\mathbf{K}_\mathbf{I}'^*) & \\ & (\mathbf{K}_\mathbf{I}''^*) \end{bmatrix}$$

The concrete relations for these submatrixes show that there are following independent oscillations classes:

I subspace (matrix $\mathbf{K}_\mathbf{I}''^*$):

-angular oscillations of solar gear S and epicycle Ep in phase + satellites St_i oscillation along axis x^* in phase),

II subspace(matrix $\mathbf{K}_\mathbf{I}''^*$):

-angular oscillations of solar gear S and epicycle Ep in anti phase + satellites St $_i$ oscillation along axis y^* and around axis φ in phase.

Similarly occurs decomposition of subspaces II and matrixes $\mathbf{K}_\mathbf{II}^{*(1)}$, $\mathbf{K}_\mathbf{II}^{*(2)}$ but instead of angular oscillations *S* and Ep there are their transversal oscillation along an axis x * (or y *), and oscillation of satellites in an anti phase.

As shows the analysis of matrix $\mathbf{K}_\mathrm{I}'^*$ the oscillations of S and Ep in a phase do not depend on angular oscillation of satellites *(φ_{St})*.

From analysis of $\mathbf{K}_\mathrm{I}'^*$ and $\mathbf{K}_\mathrm{II}'^*$ we notice, that at $h_7 = h_8 = h_6 = 0$ a zero root may be arisen. This oscillation type means the free oscillations of satellites ("navigation") at angular (or translation) oscillations solar gear and epicycle.

A. Forced oscillations. The choice of exited forces according one of these oscillation types do not induce the other oscillation types because they are orthogonal each other. The exciting forces acting in subspace II provide an independence of symmetric and anti symmetric oscillations S and Ep if they are applied simultaneously to *S* and Ep and are equal on value. Then transformation of external forces $\mathbf{F}^*$ looks like, submitted in table.

Table. The distribution of external forces

$$\mathbf{F} = \mathbf{g}_2''\mathbf{F}'' =$$

subspace	$\mathbf{K}'_{\mathbf{II}}$	$\mathbf{K}''_{\mathbf{II}}$
x'_S	$\alpha_2 f_1 h_1$	0
y'_S	$\alpha_3 f_1 h_1$	0
x'_{St}	$-2 f_1 h_1 \cos\gamma$	0

And this distribution of external forces do not induce the anti symmetric oscillations classes.

VI. Conclusions

There was established that in planetary reducer gear due to its symmetry the decomposition of oscillations arised. There are independent oscillations, such as:
-angular oscillations of solar gear and epicycle + satellites oscillations in phase;
-transversal oscillations of solar gear and epicycle + satellites oscillation in anti phase.

At equality of partial frequencies of solar gear and epicycle their oscillations in a phase do not depend on angular oscillation of satellites. By a specific parameters choice the free oscillations of satellites ("navigation") are independent from angular oscillations of solar gear and epicycle.

The choice of exited forces according to one of these oscillation types induces the oscillation of this type only because oscillation types are orthogonal to each other.

These results are correct for planetary reducer gear at any parameter change within given symmetry type.

References

[1] Airapetov E. Dynamics pf planetary mechanisms. M. Nauka, 1980

[2] Hammermesh M. Group theory and its application to physical problem. Addison -Wesley, Reading, Mass., 1962.

[3] Fulton w., Harris J. Represantation theory. A first course. Springer-Verlag, 1991.

[4] G.D.James. The representation theory of the symmetric groups, Lecture Notes in Math, vol.682, Springer-Verlag- New-York-Berlin, 1978

[5] Banakh L, Gadjieva E. Research of dynamics of regular and quasi regular systems with rotary symmetry. Mashinovedenie , 1984, N 3, C.9-16 (in Russian)

[6] Banakh L., Grinkevitch V., Ovchinnikova N.,.Fedoseev Yu. Analysis of dynamics for mechanisms with help of symmetry groups. In Mashinovedenie, 1986, N 2, P. 67-70, (in Russian)

Appendix

$$\mathbf{K}_S = \begin{bmatrix} a_1 & & \\ & a_1 & \\ & & a_2 \end{bmatrix}, \mathbf{K}_Э = \begin{bmatrix} b_1 & & \\ & b_1 & \\ & & b_2 \end{bmatrix}$$

$$\mathbf{K}_{SC_i} = \begin{bmatrix} -\alpha_i h_1 \cos\gamma & \alpha_i h_1 \sin\gamma & -r_1 \alpha_i h_1 \\ -\beta_i h_1 \cos\gamma & \beta_i h_1 \sin\gamma & -r_1 \beta_i h_1 \\ r_1 h_1 \cos\gamma & -r_1 h_1 \sin\gamma & -r_1 r_2 h_1 \end{bmatrix}$$

$$\mathbf{K}_{ЭC_i} = \begin{bmatrix} -\alpha_i h_2 \cos\gamma & -\alpha_i h_2 \sin\gamma & r_3 \alpha_i h_2 \\ -\beta_i h_2 \cos\gamma & -\beta_i h_2 \sin\gamma & -r_3 \beta_i h_2 \\ r_2 h_2 \cos\gamma & r_2 h_2 \sin\gamma & -r_3 r_2 h_2 \end{bmatrix}$$

$$\alpha_i = \cos\frac{2\pi(i-1)}{n}, \beta_i = \sin\frac{2\pi(i-1)}{n} \ (i = 1...n)$$

$$\alpha_1 = h_1 + h_7 \frac{2}{3}, \alpha_2 = h_1 r_1^2 + h_{12} \frac{1}{3}$$

$$b_1 = h_2 + h_9 \frac{4}{3}, \ b_2 = h_2 r_2^2 + h_9 \frac{4}{3} r_2^2$$

$$\begin{matrix} \mathbf{K}_{C_1} \\ \mathbf{K}_{C_2} \\ \mathbf{K}_{C_3} \end{matrix} = \begin{bmatrix} (h_1 + h_2) Cos^2\gamma + h_6 & (h_1 - h_2)\cos\gamma \sin\gamma & (h_1 - h_2) r_3 Cos\ \gamma \\ & (h_1 + h_2)\sin^2\gamma + h_6 & (h_1 + h_2) r_3 \sin\gamma \\ symm. & & (h_1 + h_2) r_3^2 \end{bmatrix}$$

The paper is supported by the RFFI project
№ 05-01-08062-офи_а.