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Efficiency enhancement in sheet metal forming analysis with a mesh regularization method

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Abstract

This paper newly proposes a mesh regularization method for the enhancement of the efficiency in sheet metal forming analysis. The regularization method searches for distorted elements with appropriate searching criteria and constructs patches including the elements to be modified. Each patch is then extended to a three-dimensional surface in order to obtain the information of the continuous coordinates. In constructing the surface enclosing each patch, NURBS (non-uniform rational B-spline) surface is employed to describe a three-dimensional free surface. On the basis of the constructed surface, each node is properly arranged to form unit elements as close as to a square. The state variables calculated from its original mesh geometry are mapped into the new mesh geometry for the next stage or incremental step of a forming analysis. The analysis results with the proposed method are compared to the results from the direct forming analysis without mesh regularization in order to confirm the validity of the method.

Keywords: Mesh regularization; Distorted element; NURBS; Patch; Finite element analysis

1. Introduction

Numerical simulation of sheet metal forming processes enjoys its prosperity with a burst of development of the computers and the related numerical techniques. The numerical analysis has extended its capabilities for sheet metal forming of complicated geometry models and multi-stage forming. In the case of a complicated geometry model, however, severe local deformation occurs to induce the increase of the computing time and deteriorate the convergence of the analysis. Distortion and severe deformation of the mesh geometry has an effect on the quality of forming analysis results especially in the case of multi-stage forming analysis when the mesh geometry formed by the forming analysis at the first stage is used for the forming analysis at the next stage. This ill behavior of the distorted mesh can be avoided by the reconstruction of the mesh system such as the total or the adaptive remeshing techniques. The adaptive remeshing technique is known to be an efficient method to reduce distortion of element during the simulation, but it still needs tremendous computing and puts restrictions among subdivided elements.

Effective methods to construct a mesh system have been proposed by many researchers. Typical methods could be *r*-method [1] in which nodal points are properly rearranged without the change of the total degrees of freedom of the mesh system, *h*-method [2] in which the number of meshes is increased with elements of the same degrees of freedom, and *p*-method [3] in which the total degrees of freedom of the mesh system is increased to enhance the accuracy of solutions. Sluiter and Hansen [4] and Talbert and Parkinson [5] constructed the analysis domain as a continuous loop and created elements in sub-loops divided from the main loop. Lo [6] constructed triangular elements in the whole domain and then constructed rectangular elements by combining adjacent triangular elements.

In this paper, a mesh regularization method is newly proposed in order to enhance the efficiency of finite element analyses of sheet metal forming. The mesh regularization method automatically finds out distorted elements with searching criteria proposed and composes patches to be modified. Each patch is then extended to three-dimensional surfaces in order to obtain the information of the continuous coordinates on the three-dimensional surface. The surface enclosing each patch is described as a three-dimensional free surface with the use of NURBS (non-uniform rational B-spline). On the basis of the constructed surface, each node is properly arranged to compose regular elements close to a square. The state variables calculated from its original mesh

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geometry are mapped into the new mesh geometry for the forming analysis at the next stage. Numerical results confirm the efficiency of the proposed method and the accuracy of the result. It is also noted that the present method is effective in the crash analyses of sheet metal members obtained from the forming simulation.

2. Regularization of the distorted element

The regularization procedure to modify distorted elements is introduced in order to enhance the efficiency of analysis for the next finite element calculation. The distorted elements are selected with appropriate searching criteria and allocated to several patches for regularization. The patches are extended to three-dimensional surfaces with the use of NURBS for full information of the continuous coordinates on the three-dimensional surface. On obtaining the new coordinates of each node, the distorted elements are regularized to a regular element that is close to a square.

2.1. The criterion of mesh distortion

Distorted meshes are selected with the two geometrical criteria: one is the inner angle; and the other is the aspect ratio of the element side.

2.1.1. Inner angle

The inner angle of a quadrilateral element should be close to the right angle for good results from finite element calculation. Zhu et al. [7] defined the reasonable element when the four inner angles are formed with the angle of $90^\circ \pm 45^\circ$ while Lo and Lee [8] proposed the inner angle of $90^\circ \pm 52.5^\circ$ as the same criterion. The criterion of mesh distortion for the inner angle is determined by constituting Eq. (1). A mesh is regarded as distorted when Eq. (1) is less than $\pi/3$ or $(\delta\theta_i)_{\max}$ in Eq. (3) [9] is greater than $\pi/6$. The criterion is rather strict in order to avoid the geometrical limitation in case of applying the regularization method in confined regions:

$$\vec{f}_Q = \delta\theta_1\hat{e}_1 + \delta\theta_2\hat{e}_2 + \delta\theta_3\hat{e}_3 + \delta\theta_4\hat{e}_4 \tag{1}$$

$$\|\vec{f}_Q\| = \sqrt{\sum_{i=1}^{4}(\delta\theta_i)^2} \tag{2}$$

$$\delta\theta_i = \left|\tfrac{1}{2}\pi - \theta_i\right| \tag{3}$$

2.1.2. Aspect ratio of the element

The ideal aspect ratio of the element side should be unity when the four sides of an element have the same length. The aspect ratio is defined as Eq. (4) and then the distortion is defined when it is less than 5 that could be much less for a strict criterion:

$$\frac{\max\{r_{12}, r_{23}, r_{34}, r_{41}\}}{\min\{r_{12}, r_{23}, r_{34}, r_{41}\}} \tag{4}$$

where r_{ij} is the length of each element side.

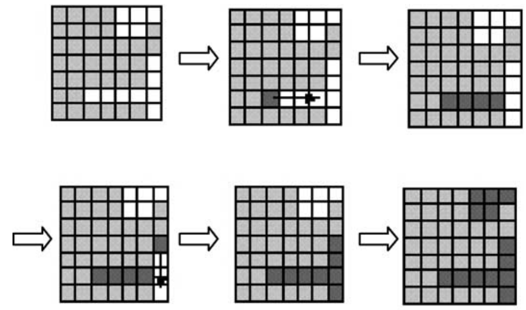

Fig. 1. Process for construction of a patch.

2.2. Domain construction

2.2.1. Construction of the patch

Distorted elements selected by the criteria of mesh distortion are distributed in various regions according to the complexity of the shape of formed geometry. These elements are allocated to patches constructed for the efficiency of the algorithm. The shape of patches is made up for rectangular shapes including all distorted elements for expanding the region of regularization and applying to NURBS surface explained in next section. This procedure is shown in Fig. 1. When holes and edges are located between distorted elements, the regions are filled up to make patches a rectangular shape.

The patch is then mapped to a three-dimensional free surface by using NURBS surface. The procedure is important to obtain entire information of the continuous coordinates on the three-dimensional surface. NURBS surface can describe the complex shape quickly by using less data points and does not change the entire domain data due to the local change.

2.2.2. NURBS surface

NURBS surface is generally expressed by Eq. (5) as the p-order in the u-direction and the q-order in the v-direction [10]:

$$S(u, v) = \frac{\sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,p}(u)N_{j,q}(v)w_{i,j}P_{i,j}}{\sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,p}(u)N_{j,q}(v)w_{i,j}} \tag{5}$$

where $P_{i,j}$ is the control points as the u-, v-direction, $w_{i,j}$ the weight factor and $N_{u,p}(u)$, $N_{j,q}(u)$ the basis function that are expressed by Eq. (6):

$$N_{i,0} = \begin{cases} 1 & \text{if } u_i \le u \le u_{i+1}, \\ 0 & \text{otherwise,} \end{cases}$$
$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u) \tag{6}$$

In order to map the nodes from the patches onto the constructed surface, a number of points are created for their coordinates on the NURBS surface. The location of each

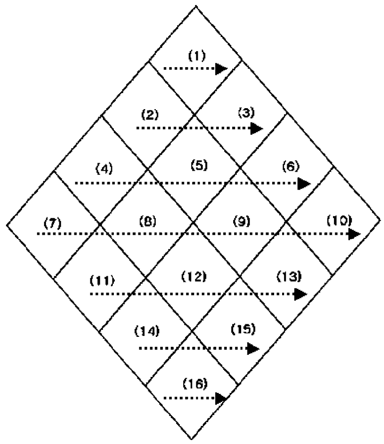

Fig. 2. Selecting direction of distorted elements.

moving node by applying a regularization method is determined such that the location of a point has the minimum distance between nodes on NURBS surface. The information on the coordinates of the nodal points to be moved is stored to construct a new mesh system.

2.3. Regularization procedure

The regularization method is carried out with the unit of a patch that forms a rectangular shape. Finite elements to be regularized is selected by the order of Fig. 2. Each selected element is divided by two triangular elements and then the divided element is made of a right triangular element by relocating the vertex on the circle having the diameter from $\vec{x}_1$ to $\vec{x}_2$ as shown in Eq. (7) and Fig. 3. When the procedure terminates, the same procedure is repeated in the opposite direction:

$$\frac{\vec{x}_1+\vec{x}_2}{2}=\vec{x}_{\mathrm{cen}},\qquad \frac{|\vec{x}_1-\vec{x}_2|}{2}=r,\qquad \vec{x}_{\mathrm{cur}}-\vec{x}_{\mathrm{cen}}=\vec{x}_{\mathrm{dir}},$$
$$\vec{x}_{\mathrm{new}}=\frac{\vec{x}_{\mathrm{dir}}}{|\vec{x}_{\mathrm{dir}}|}r\times \mathrm{factor}+\vec{x}_{\mathrm{cen}} \tag{7}$$

The final location of a node relocated by using the regularization method is substituted for the location of a point on NURBS surface. After the regularization procedure is finished, a simple soothing procedure is carried out by Eq. (8) for the rough region generated during the procedure:

$$P_{\mathrm{N}}=\frac{\sum_{i=1}^{N}A_iC_i}{\sum_{i=1}^{N}A_i} \tag{8}$$

where P_{N} is the coordinate of a new node, A_i the areas of adjacent elements and C_i the centroid of the adjacent elements.

2.4. Level of distortion

As a distortion factor, level of distortion (LD) is newly proposed. LD can be used to evaluate the degree of improvement in the element quality:

$$\mathrm{LD}=A\times B \tag{9}$$

where

$$A=\frac{\sum_{i=1}^{4}|\sin\theta_i|}{4},\qquad B=\tanh\left(k\times B'\right) \tag{10}$$

$$B'=\frac{\min\{r_{12},r_{23},r_{34},r_{41}\}}{\max\{r_{12},r_{23},r_{34},r_{41}\}},\qquad k=\frac{\tanh^{-1}(\beta)}{\alpha} \tag{11}$$

LD has the value between 0 and 1; when LD = 1, the element is an ideal element of a square and when LD = 0, the quadrilateral element becomes a triangular element. θ_i are the four inner angles of an element, so A is the factor for the inner angle. B is the factor for the aspect ratio of element sides and is defined by the hyperbolic tangent function in order to make LD less sensitive to the change of B. For example, when the reasonable aspect ratio of the element side is 1:4, the value of B can be adjusted by applying $\alpha = 0.25$ and $\beta = 0.6$ such that the slope of the function B is changed abruptly around the value of $B' = 0.25$. Consequently, the value of LD decreases rapidly when the aspect ratio B' is less than 0.25 while the value of LD increases slowly when the B' is greater than 0.25. This scheme can regulate the inner angle and the aspect ratio to have the equal effect on the LD.

2.5. Mapping of the state variables

When the regularized mesh system is used for the next calculation of the forming analysis or the structural analysis, mapping of the state variables is needed for more accurate analysis considering the previous forming history. The mapping procedure is to map the calculated state variables in the original mesh system onto the regularized mesh system. As shown in Fig. 4, a sphere is constructed surrounding a new node such that the state variables of nodes in the sphere have an effect on the state variables of the new node. The state variables of the new node are determined from the state variables of the neighboring nodes in the sphere by imposing the weighting factor inversely proportional to the distance between the two nodes as shown in Eq. (12):

$$V_{\mathrm{c}}=\frac{\sum_{j=1}^{m}V_j/r_j}{\sum_{i=1}^{m}1/r_i} \tag{12}$$

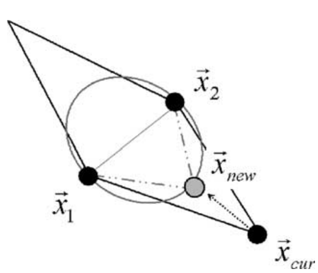

Fig. 3. Regularization scheme by moving nodes.

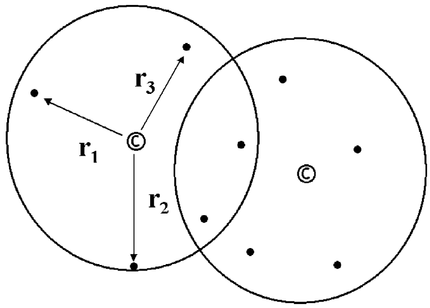

Fig. 4. Control sphere for mapping of the state variables.

where V_j is the state variable calculated on the original mesh system, and r_j the distance between the new node and the neighboring nodes.

3. Numerical examples

3.1. Forming analysis of an oil pan

While oil pans are usually fabricated with a two-stage process in the press shop, the present analysis is carried out with a single-stage process as shown in Fig. 5 that describes the punch and die set.

The regularization method can be applied to the finite element mesh system whenever needed for enhancement of the computation efficiency. In this example for demonstration, the method is applied to the analysis of oil pan forming at two forming intervals for regularization of distorted meshes as directed in Fig. 6.

Fig. 7 explains the procedure of the regularization method. Fig. 7(a) shows the deformed shape at the punch stroke of

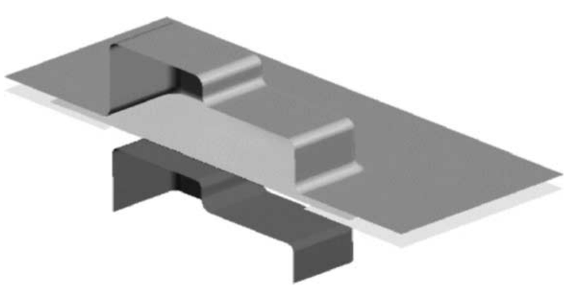

Fig. 5. Punch and die set for oil pan forming.

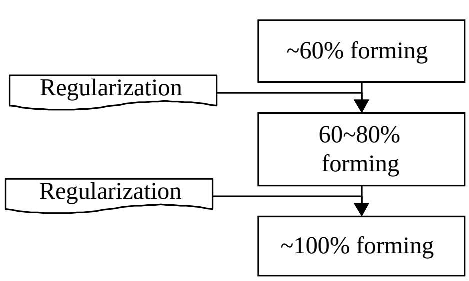

Fig. 6. Applying the regularization method to the forming analysis.

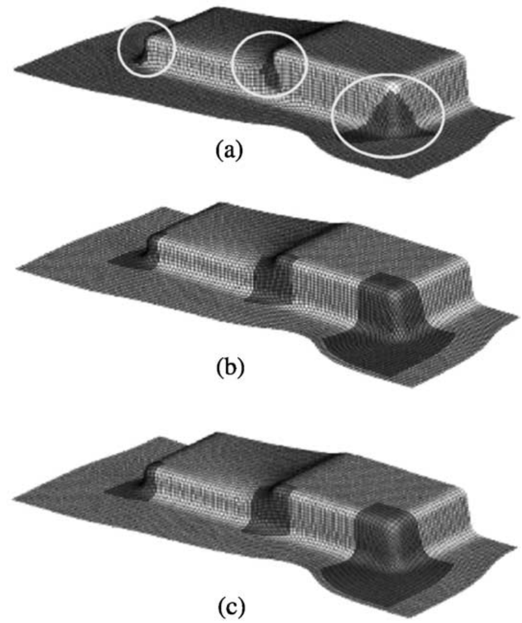

Fig. 7. Procedure of regularization: (a) searching distorted elements; (b) constructing patches for distorted elements; (c) regularization of distorted elements.

60% and three parts of mesh distortion by the forming procedure. It indicates that the number of patches to be constructed is 3. Distorted meshes are selected according to the two geometrical criteria for mesh distortion. And then the patches of a rectangular shape are formed to include all distorted elements as shown in Fig. 7(b). Finally, the elements in the patches are regularized as shown in Fig. 7(c).

In order to evaluate the degree of improvement in the element quality after applying the regularization method, the value of LD for the regularized mesh system is compared the one for the original mesh system. The LD values for the regularized mesh system have uniform distribution throughout the elements while those for the original mesh system have wide variation as shown in Fig. 8. It means that the quality of the regularized mesh system is enhanced with the same level distortion. Consequently, explicit finite element computation with the regularized mesh system can be preceded with a larger incremental time step as shown in Fig. 9. In this analysis of oil pan forming, the computing time with the regularized mesh system is reduced about 12% even after two times of regularization. The amount of reduction in the computing time can be increased with more frequent regularization.

3.2. Crash analysis of a front side member

The crash analysis is usually carried out without considering the forming effect and adopts the mesh system apart form the forming analysis. In case that the forming effect is considered to improve the accuracy and reliability of the analysis results, the mesh system for the forming analysis could be directly used in the crash analysis for the efficiency of the analysis. The mesh system after the forming analysis,

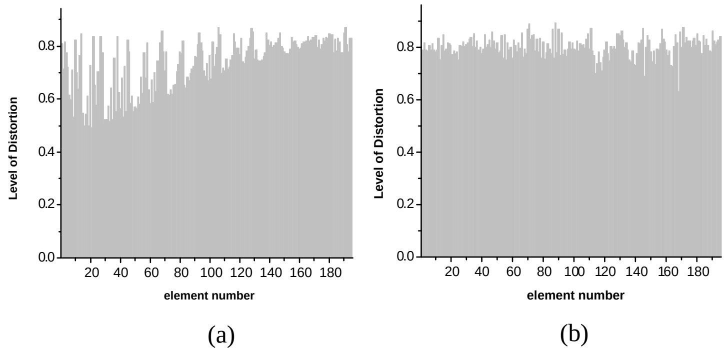

Fig. 8. Comparison of LD: (a) original mesh; (b) regularized mesh.

however, has too many distorted meshes due to severe deformation and distortion be used directly in the crash analysis without remeshing. One remedy is to construct a new mesh system and the other is to modify the mesh system after the forming analysis. The latter can be very efficient if the remeshing process is successfully carried out. For an efficient remeshing process, the regularization method can be applied to transform distorted meshes into a near square. In this example, a part of the front side member, named 'front reinforcement', in Fig. 10 is selected for the crash analysis. The irregular finite elements in the local distorted region of the member after forming analysis are modified to a regular element using the regularization method as shown in Fig. 11 and then the regularized mesh system is used in the crash analysis. The analysis condition is depicted in Fig. 12.

The crash analysis with the regularized mesh system could be carried out with a lager time step as shown in Fig. 13 without sacrificing the accuracy of the analysis result. The computing time for the crash analysis was reduced by 40% of the time with the original mesh system. The analysis result was encouraging in both the computing time and the accuracy of the computation result and proved that the regularized mesh system could be used effectively to enhance the efficiency of the numerical analysis.

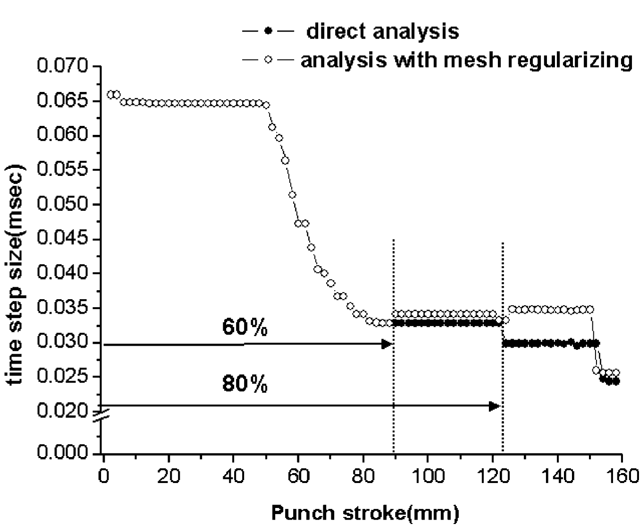

Fig. 9. Comparison of the time step size with respect to the punch stroke.

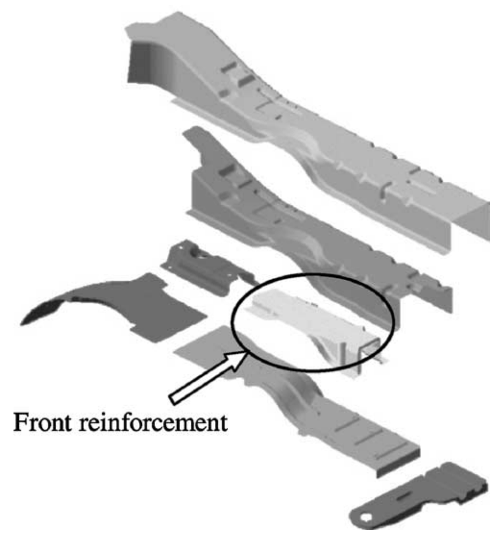

Fig. 10. Assembly of a front side member.

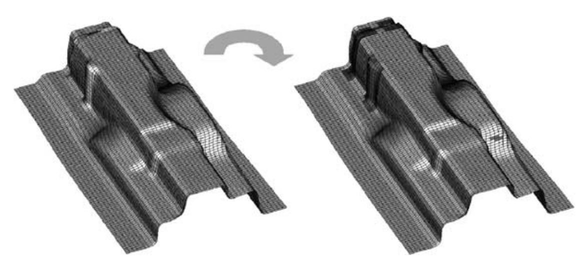

Fig. 11. Regularization of a front reinforcement.

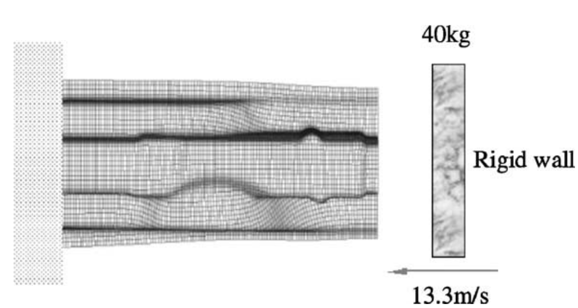

Fig. 12. Condition of the impact analysis.

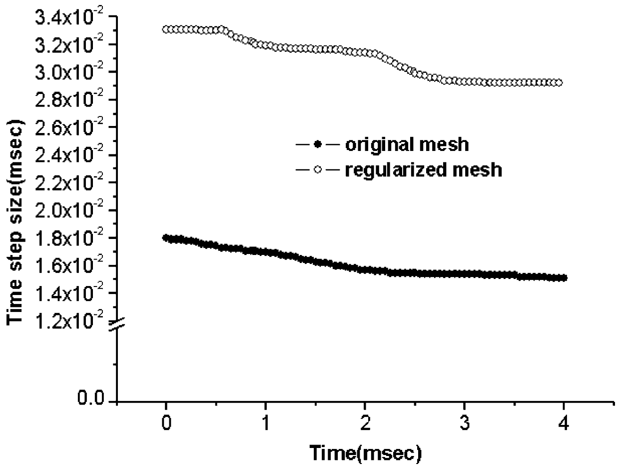

Fig. 13. Comparison of the time step size between the original and regularized mesh system.

4. Conclusion

A mesh regularization method is newly proposed in order to enhance the efficiency of finite element analyses of sheet metal forming. Meshes in the sheet metal forming analysis are distorted so severely that the subsequent analysis would be difficult or produce poor results. This can be avoided by the present method with the minimum effort of remeshing. Mesh regularization can be carried out during the incremental analysis or for the next stage in multi-stage forming. It is also proved that the analysis efficiency is greatly improved when mesh regularization is carried out for the crash analysis of formed members obtained from the sheet metal forming simulation. Numerical results confirm the validity and efficiency of the proposed method as well as the accuracy of the result.

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