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An analytical design for three circular-arc cams

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Abstract

In this paper we have presented an analytical description for three circular-arc cam profiles. An analytical formulation for cam profiles has been proposed and discussed as a function of size parameters for design purposes. Numerical examples have been reported to prove the soundness of the analytical design procedure and show the engineering feasibility of suitable three circular-arc cams.

1. Introduction

A cam is a mechanical element, which is used to transmit a desired motion to another mechanical element by direct surface contact.

Generally, a cam is a mechanism, which is composed of three different fundamental parts from a kinematic viewpoint [1,2]: a cam, which is a driving element; a follower, which is a driven element and a fixed frame. Cam mechanisms are usually implemented in most modern applications and in particular in automatic machines and instruments, internal combustion engines and control systems [3].

Cam and follower mechanisms can be very cheap, and simple. They have few moving parts and can be built with very small size.

The design of cam profile has been based on simply geometric curves, [4], such as: parabolic, harmonic, cycloidal and trapezoidal curves [2,5] and their combinations [1,2,6,7].

In this paper we have addressed attention to cam profiles, which are designed as a collection of circular arcs. Therefore they are called circular-arc cams [5,8].

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Circular-arc cams can be easily machined and can be used in low-speed applications [9]. In addition, circular-arc cams could be used for micro-mechanisms and nano-mechanisms since very small manufacturing can be properly obtained by using elementary geometry.

An undesirable characteristic of this type of cam is the sudden change in the acceleration at the profile points where arcs of different radii are joined [5].

A limited number of circular-arcs is usually advisable so that the design, construction and operation of cam transmission can be not very complicated and they can become a compromise for simplicity and economic characteristics that are the basic advantages of circular-arc cams [8].

Recently new attention has been addressed to circular-arc cams by using descriptive viewpoint [10], and for design purposes [11,12].

In this paper we have described three circular-arc cams by taking into consideration the geometrical design parameters. An analytical formulation has been proposed for three circular-arc cams as an extension of a formulation for two circular-arc cams that has been presented in a previous paper [12].

2. An analytical model for three circular-arc cams

An analytical formulation can be proposed for three circular-arc cams in agreement with design parameters of the model shown in Figs. 1 and 2.

Significant parameters for a mechanical design of a three circular-arc cam are: Fig. 1 [8]; the rise angle α_s, the dwell angle α_r, the return angle α_d, the action angle $\alpha_a = \alpha_s + \alpha_r + \alpha_d$, the maximum lift h_1.

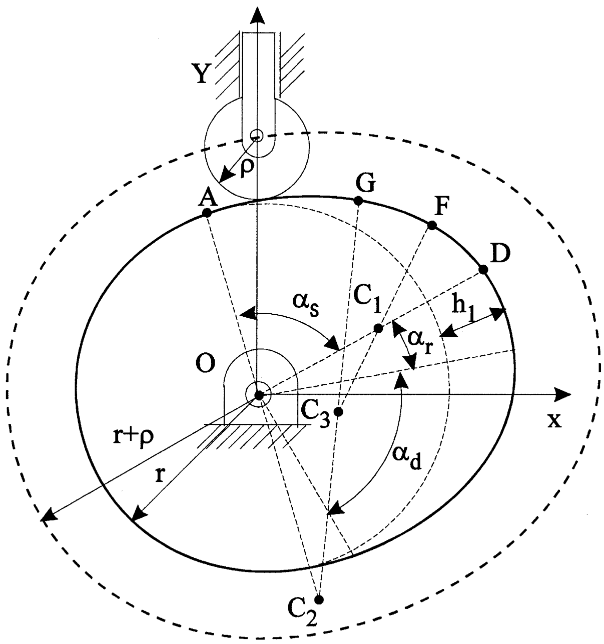

Fig. 1. Design parameters for general three circular-arc cams.

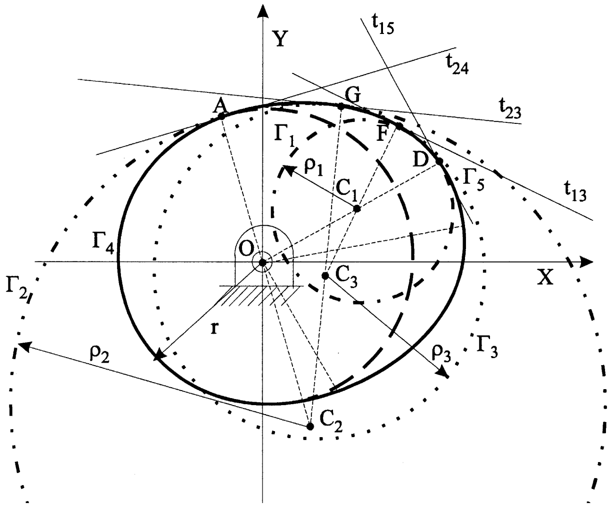

Fig. 2. Characteristic loci for three circular-arc cams.

The characteristic loci of a three circular-arc cams are shown in Fig. 2 as: the first circle Γ_1 of the cam profile with ρ_1 radius and centre C_1; the second circle Γ_2 of the cam profile with ρ_2 radius and centre C_2; the third circle Γ_3 of the cam profile with ρ_3 radius and centre C_3; the base circle Γ_4 with radius r and the centre is O; the lift circle Γ_5 of the cam profile with $(r + h_1)$ radius and centre O; the roller circle with radius ρ centred on the follower axis. In addition significant points are: $D \equiv (x_D; y_D)$ which is the point joining Γ_1 with Γ_5; $F \equiv (x_F; y_F)$ which is the point joining Γ_1 with Γ_3; $G \equiv (x_G; y_G)$ which is the point joining Γ_3 with Γ_2; $A \equiv (x_A; y_A)$ which is the point joining Γ_2 with Γ_4. x and y are Cartesian co-ordinates of points with respect to the fixed frame OXY, whose origin O is a point of the cam rotation axis. Additional significant loci are: $\boldsymbol{t}_{13}$ which is the coincident tangential vector between Γ_1 and Γ_3; $\boldsymbol{t}_{15}$ which is the coincident tangential vector between Γ_1 and Γ_5; $\boldsymbol{t}_{23}$ which is the coincident tangential vector between Γ_2 and Γ_3; $\boldsymbol{t}_{24}$ which is the coincident tangential vector between Γ_2 and Γ_4.

The model shown in Figs. 1 and 2 can be used to deduce a formulation, which can be useful both for characterizing and designing three circular-arc cams. Analytical description can be proposed when the circles are formulated in the suitable form:

- circle Γ_1 with radius $\rho_1^2 = (x_1 - x_F)^2 + (y_1 - y_F)^2$ passing through point F as

$$x^2 + y^2 - 2xx_1 - 2yy_1 - x_F^2 - y_F^2 + 2x_1x_F + 2y_1y_F = 0 \tag{1}$$

- circle Γ_2 with radius $\rho_2^2 = (x_2 - x_A)^2 + (y_2 - y_A)^2$ passing through point A as

$$x^2 + y^2 - 2xx_2 - 2yy_2 - x_A^2 - y_A^2 + 2x_2x_A + 2y_2y_A = 0 \tag{2}$$

- circle Γ_2 with radius $\rho_2^2 = (x_2 - x_G)^2 + (y_2 - y_G)^2$ passing through point G as

$$x^2 + y^2 - 2xx_2 - 2yy_2 - x_G^2 - y_G^2 + 2x_2x_G + 2y_2y_G = 0 \tag{3}$$

- circle Γ_3 with radius $\rho_3^2 = (x_3 - x_F)^2 + (y_3 - y_F)^2$ passing through point F as
$$x^2 + y^2 - 2xx_3 - 2yy_3 - x_F^2 - y_F^2 + 2x_3x_F + 2y_3y_F = 0 \quad (4)$$
- circle Γ_3 with radius $\rho_3^2 = (x_3 - x_G)^2 + (y_3 - y_G)^2$ passing through point G as
$$x^2 + y^2 - 2xx_3 - 2yy_3 - x_G^2 - y_G^2 + 2x_3x_G + 2y_3y_G = 0 \quad (5)$$
- circle Γ_4 with radius r as
$$x^2 + y^2 = r^2 \quad (6)$$
- circle Γ_5 with radius $(r + h_1)$ as
$$x^2 + y^2 = (r + h_1)^2 \quad (7)$$

Additional characteristic conditions can be expressed in the form as

- the first circle Γ_1 and lift circle Γ_5 must have the same tangential vector $\boldsymbol{t}_{15}$ at point D expressed as
$$xx_1 + yy_1 - x_1x_D - y_1y_D = 0 \quad (8)$$
- the base circle Γ_4 and second circle Γ_2 must have the same tangential vector $\boldsymbol{t}_{24}$ at point A expressed as
$$xx_2 + yy_2 - x_2x_A - y_2y_A = 0 \quad (9)$$
- the second circle Γ_2 and third circle Γ_3 must have the same tangential vector $\boldsymbol{t}_{23}$ at point G expressed as
$$x(x_3 - x_2) + y(y_3 - y_2) + x_3x_G + y_3y_G - x_1x_G - y_1y_G = 0 \quad (10)$$
- the first circle Γ_1 and the second circle Γ_2 must have the same tangential vector $\boldsymbol{t}_{12}$ at point F expressed as
$$x(x_1 - x_3) + y(y_1 - y_3) + x_3x_F + y_3y_F - x_1x_F - y_1y_F = 0 \quad (11)$$

Eqs. (1)–(11) may describe a general model for three circular-arc cams and can be used to draw the mechanical design as shown in Fig. 2.

3. An analytical design procedure

Eqs. (1)–(11) can be used to deduce a suitable system of equations, which allows solving the co-ordinates of the points C_1, C_2, C_3, F and G when suitable data are assumed.

It is possible to distinguish four different design cases by using the proposed analytical description.

In a first case we can consider that the numeric value of the parameters h_1, r, α_s, α_r, α_d, ρ_1, ρ_2, and co-ordinates of the points A, C_1, C_2, D and G are given, and the co-ordinates of points C_3, F are the unknowns. When the action angle α_a is equal to 180°, the co-ordinate x_A of point A is equal to zero. Since A is the point joining Γ_2 and Γ_4 then the centre C_2 of the second circle Γ_2 lies on the Y axis and therefore the co-ordinate x_2 of the centre C_2 is equal to zero. By using Eqs. (1)–(11) it is possible to deduce a suitable system of equations which allows to solve the co-ordinates of the points C_3 and F. Analytical formulation can be expressed by means of the following conditions:

- the first circle Γ_1 passing across points F and D in the form

$$(x_F - x_1)^2 + (y_F - y_1)^2 = (x_D - x_1)^2 + (y_D - y_1)^2 \tag{12}$$

- the third circle Γ_3 passing across points F and G in the form

$$(x_F - x_3)^2 + (y_F - y_3)^2 = (x_G - x_3)^2 + (y_G - y_3)^2 \tag{13}$$

- coincident tangents to Γ_1 and Γ_3 at the point F in the form

$$\frac{x_3 - x_1}{y_3 - y_1} = \frac{x_F - x_3}{y_F - y_3} \tag{14}$$

- coincident tangents to Γ_2 and Γ_3 at the point G in the form

$$\frac{x_2 - x_3}{y_2 - y_3} = \frac{x_G - x_2}{y_G - y_2} \tag{15}$$

When $x_2 = x_A = 0$ are assumed, Eqs. (12)–(15) can be expressed as

$$\begin{aligned}
&x_F^2 + y_F^2 - 2x_1x_F - 2y_1y_F - x_D^2 - y_D^2 + 2x_1x_D + 2y_1y_D = 0\\
&x_F^2 + y_F^2 - 2x_3x_F - 2y_3y_F - x_G^2 - y_G^2 + 2x_3x_G + 2y_3y_G = 0\\
&(x_F - x_3)(y_3 - y_1) - (x_3 - x_1)(y_F - y_3) = 0\\
&x_G(y_2 - y_3) - x_3(y_G - y_2) = 0
\end{aligned} \tag{16}$$

If the position of the centre C_2 is unknown and the direction of the centre C_1 lies on the OD straight line, we can approach referring to Fig. 2 a second problem: namely the value of the parameters h_1, r, α_s, α_r, α_d, ρ_1, and the co-ordinates of the points C_2, A, D and G are known and the co-ordinates of the points C_1, F and C_3 are unknown. Again we may assume $\alpha_a = 180°$ and consequently $x_A = x_2 = 0$. Two additional conditions are necessary to have a solvable system together with Eq. (9). They are

- the second circle Γ_2 passing across points G and A in the form

$$(x_G - x_2)^2 + (y_G - y_2)^2 = (x_A - x_2)^2 + (y_A - y_2)^2 \tag{17}$$

- straight-line containing points O, A and C_2 in the form

$$x_2y_A - x_Ay_2 = 0 \tag{18}$$

Thus, the second case can be solved by Eqs. (16)–(18).

If the position of the centre C_1 is unknown but we know that it lies on the OD straight line, we can approach a third design problem: namely the value of the parameters h_1, r, α_s, α_r, α_d, ρ_1, and the co-ordinates of the points A, D and G are known and the co-ordinates of the points C_1, C_2, F and C_3 are unknown. Again we may assume $\alpha_a = 180°$ and consequently $x_A = x_2 = 0$. Two additional conditions are necessary to have a solvable system together with Eqs. (16)–(18). They are

- the first circle Γ_1 passing across point D in the form

$$(x_D - x_1)^2 + (y_D - y_1)^2 = \rho_1^2 \tag{19}$$

- straight-line containing points O, D and C_1 in the form

$$x_D y_1 - x_1 y_D = 0 \tag{20}$$

Finally we may approach the fourth case when $\alpha_a < 180°$ and $x_A \neq 0$ and also $x_2 \neq 0$. Referring to Fig. 1, in which α_a is the angle between the general position of the point A and the Y axis, the value of the parameters h_1, r, α_s, α_r, α_d, ρ_1, and the co-ordinates of the points A, D and G are known and the co-ordinates of the points C_1, C_2, C_3 and F are unknown. The fourth of Eq. (16) can be expressed as

$$(x_2 - x_3)(y_G - y_2) - (y_2 - y_3)(x_G - x_2) = 0 \tag{21}$$

Thus, the general design case can be solved by using Eqs. (12)–(14) and Eqs. (17)–(21).

A design procedure can be obtained by using the above-mentioned formulation in order to compute the design parameters. In particular, the proposed formulation has been useful for a design procedure which makes use of MAPLE to solve for the design unknowns.

4. Numerical examples

Several numeric examples have been successfully computed in order to prove the soundness and numerical efficiency of the proposed design formulation. It has been found that only one solution can represent a significant circular-arc cam design for any of the formulated design cases.

In the Example 1 of Fig. 3 referring to the first design case, the data are given as $h_1 = 15$ mm, $r = 40$ mm, $\alpha_r = 40°$, $\alpha_s = \alpha_d = 70°$, $\rho_1 = 17$ mm, $A \equiv (0; 40\ \text{mm})$, $D \equiv (51.68\ \text{mm};\ 18.81\ \text{mm})$ $C_1 \equiv (35.71\ \text{mm};\ 13.00\ \text{mm})$, $C_2 \equiv (0\ \text{mm};\ -75.64\ \text{mm})$ and $G \equiv (22.24\ \text{mm};\ 37.84\ \text{mm})$. Fig. 3 shows results for the design case, which has been formulated by Eq. (16). In particular, Fig. 3(a) shows the first solution of the analytical formulation. We can note that points F, C_1 and C_3 are aligned in the order F, C_1 and C_3 and points G, C_3 and C_2 in the order G, C_3 and C_2 respectively to the first and second arcs cam profile. Fig. 3(b) shows the second solution of the analytical formulation. A cam profile cannot be identified since F point does not lie also on circle Γ_1. Significant points F, C_1 and C_3 are aligned in the same order with respect to the case in Fig. 3(a); points G, C_2 and C_3 are aligned in the C_2, G and C_3 sequential order which is different respect to the case in Fig. 3(a) and do not give a cam profile. Fig. 3(c) shows the third solution of analytical formulation that is similar to the case of Fig. 3(b). Fig. 3(d) shows the fourth solution of analytical formulation. We can note that in correspondence of point D there is a cusp. In addition, points F and G are very near to centre C_3 so that a sudden change of curvature is obtained in the cam profile as shown in Fig. 3(d). Thus a practical feasible design is represented only by Fig. 3(a) that can be characterised by the proper order F, C_1 and C_3 and G, C_3 and C_2 of the meaningful points.

The feasible numerical solution in Fig. 3(a) is characterised by the values: $x_F = 46.78$ mm, $y_F = 25.91$ mm, $x_3 = 11.99$ mm, $y_3 = -14.47$ mm.

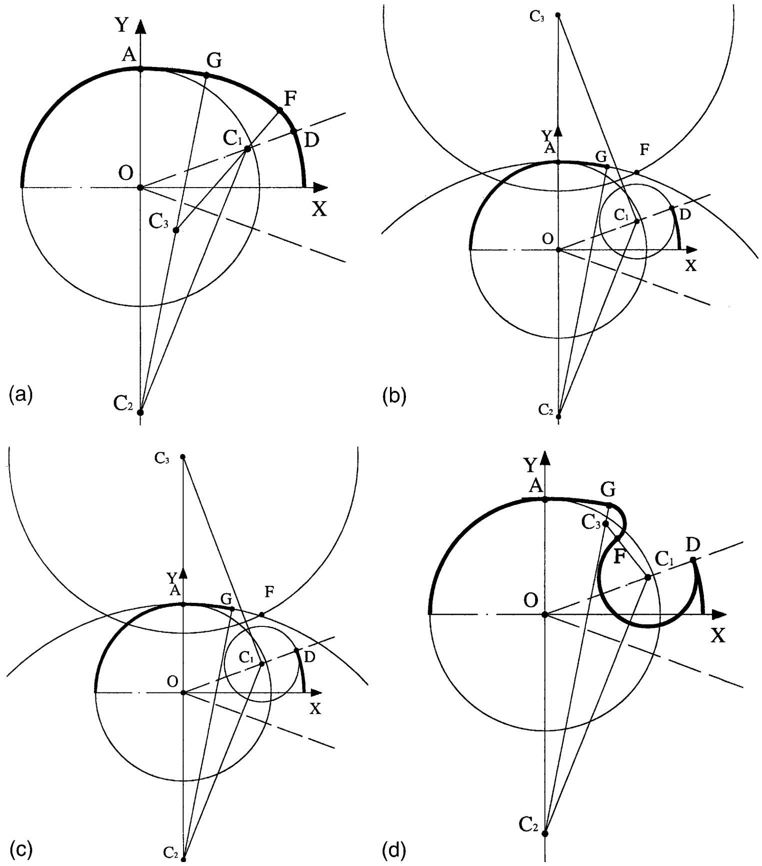

Fig. 3. Examples 1 and 2: graphical representation of design solutions for Eq. (16) and design solutions for Eqs. (16)–(18). Only case (a) is a practical feasible design.

In the Example 2 of Fig. 3 the data are given as $h_1 = 15$ mm, $r = 40$ mm, $\alpha_r = 40°$, $\alpha_s = \alpha_d = 70°$, $\rho_1 = 17$ mm, $A \equiv (0;\ 40\ \text{mm})$, $D \equiv (51.68\ \text{mm};\ 18.81\ \text{mm})$, $C_1 \equiv (35.71\ \text{mm};\ 13.00\ \text{mm})$ and $G \equiv (22.24\ \text{mm};\ 37.84\ \text{mm})$.

In this case Fig. 3 represents also the design solution which has been obtained by using Eqs. (16)–(18) for the second design case.

The feasible numerical solution is characterised by the values: $x_F = 46.78$ mm, $y_F = 25.91$ mm, $x_3 = 11.99$ mm, $y_3 = -14.47$ mm, $x_2 = 0$ mm, $y_2 = -75.64$ mm.

In the Example 3 of Fig. 4 referring to the third design case the data are given as $h_1 = 15$ mm, $r = 40$ mm, $\alpha_r = 40°$, $\alpha_s = \alpha_d = 70°$, $\rho_1 = 17$ mm, $A \equiv (0;\ 40\ \text{mm})$, $D \equiv (51.68\ \text{mm};\ 18.81\ \text{mm})$ and $G \equiv (22.24\ \text{mm};\ 37.84\ \text{mm})$.

Fig. 4 shows results for the design case, which has been formulated by Eqs. (16)–(20). Fig. 4(a) shows the first solution of analytical formulation. This case is similar to the solution represented in Fig. 3(d). Fig. 4(b) shows the second solution of analytical formulation. We can note that point F is located below point D so that points F, C_1 and C_3 are not aligned. Fig. 3(c) shows the third

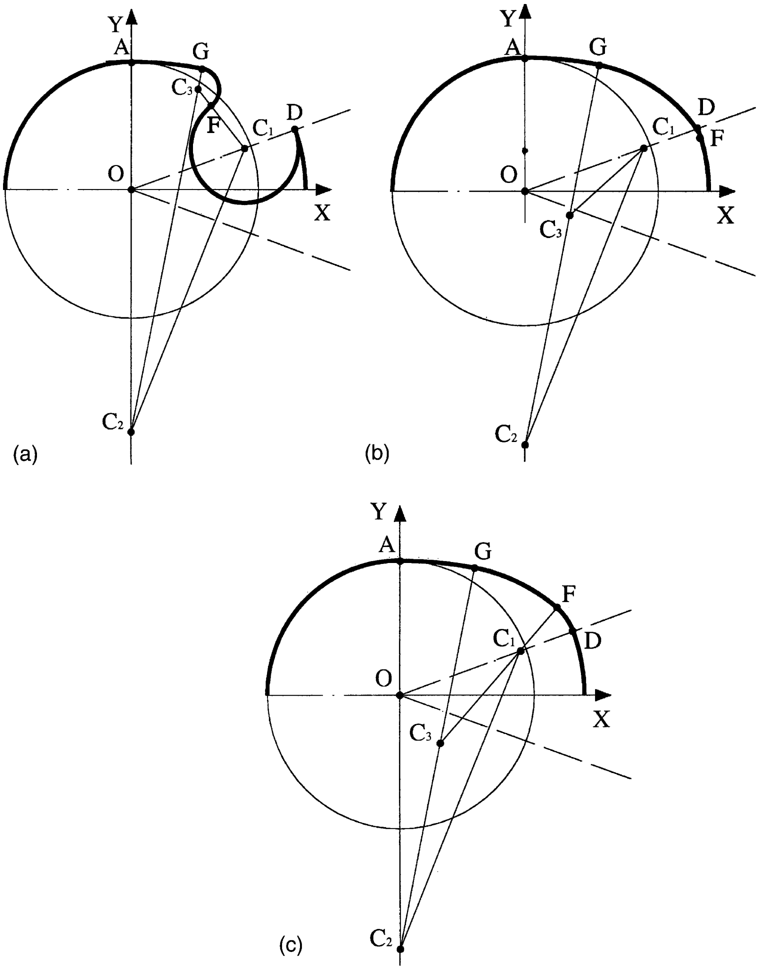

Fig. 4. Example 3: graphical representation of design solutions for Eqs. (16)–(20). Only case (c) is a practical feasible design.

solution of analytical formulation, which is the same of the case reported in Fig. 3(a). Thus a practical feasible design is represented only by Fig. 4(c).

The feasible numerical solution is characterised by the values: $x_F = 46.78$ mm, $y_F = 25.91$ mm, $x_3 = 11.99$ mm, $y_3 = -14.47$ mm, $x_2 = 0$ mm, $y_2 = -75.64$ mm, $x_1 = 35.71$ mm, $y_1 = 13.00$ mm.

In the Example 4 of Fig. 5 referring to the fourth design case, the data are given as $h_1 = 15$ mm, $r = 40$ mm, $\alpha_r = 40°$, $\alpha_s = \alpha_d = 70°$, $\rho_1 = 17$ mm, $A \equiv (3.48$ mm; 39.84 mm$)$, $D \equiv (51.68$ mm; 18.81 mm$)$ and $G \equiv (22.24$ mm; 37.84 mm$)$.

Fig. 5 shows results for the design case, which has been formulated by Eqs. (16)–(21). Fig. 5(a) shows the first solution of the analytical formulation. This design is similar to the case reported in Fig. 4(a), but the location of point C_1 is different. Points F, C_1 and C_3 are aligned in the C_3, F and C_1 order. Fig. 5(b) shows the second solution of analytical formulation, which is similar to the case in Fig. 4(a). Fig. 5(c) shows the third solution of analytical formulation. This case shows a

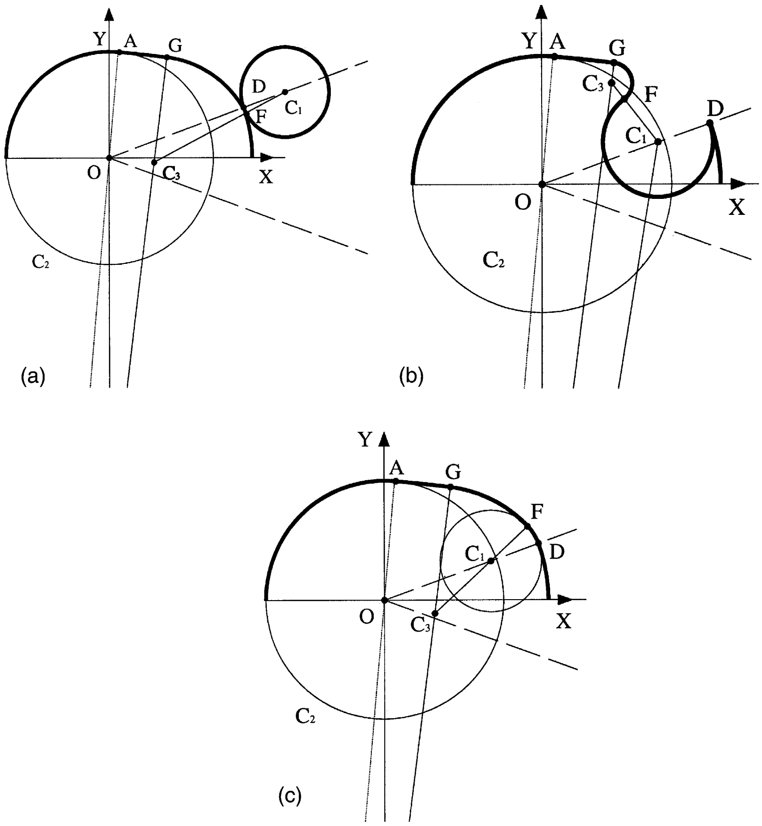

Fig. 5. Example 4: graphical representation of design solutions for Eqs. (16)–(21). Only case (c) is a practical feasible design.

solution, which is similar to the case reported in Fig. 4(c). Thus a practical feasible design is represented only by Fig. 5(c).

The feasible numerical solution is characterised by the values: $x_F = 48.15$ mm, $y_F = 24.58$ mm, $x_3 = 16.92$ mm, $y_3 = -4.50$ mm, $x_2 = -40.01$ mm, $y_2 = -457.26$ mm, $x_1 = 35.71$ mm, $y_1 = 13.00$ mm.

5. Applications

A novel interest can be addressed to approximate design of cam profiles for both new design purposes and manufacturing needs.

Analytical design formulation is required to obtain efficient design algorithms. In addition, closed-form formulation can be also useful to characterise cam profiles in both analysis procedures and synthesis criteria. The approximated profiles with circular-arcs can be of particular interest also to obtain analytical expressions for kinematic characteristics of any profiles that can be approximated by segments of proper circular arcs.

Indeed, the circular-arc cam profiles have become of current interest because of applications in mini-mechanisms and micro-mechanisms. In fact, when the size of a mechanical design is reduced to the scale of millimeters (mini-mechanisms) and even micron (micro-mechanisms) the manufacturing of polynomial cam profile becomes difficult and even more complicated is a way to verify it. Therefore, it can be convenient to design circular-arc cam profiles that can be also easily tested experimentally.

In addition, stronger and stronger demand of low-cost automation is giving new interest to approximate designs, which can be used only for specific tasks. This is the case of circular-arc cam profiles that can be conveniently used in low speed machinery or in low-precision applications.

6. Conclusions

In this paper we have proposed an analytical formulation which describes the basic design characteristics of three circular-arc cams. A design algorithm has been deduced from the formulation, which solves design problems with great numerical efficiency. Numerical examples have been reported in the paper to show and discuss the multiple design solutions and the engineering feasibility of three circular-arc cams.

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